



Experimental Study of the Convergence of the Chebyshev Method

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Abstract. This paper presents an experimental investigation of the convergence properties of the Chebyshev spectral method for solving ordinary differential equations. The method is based on approximating the target solution by an expansion in Chebyshev polynomials and applying the collocation approach at Chebyshev–Lobatto nodes. The numerical experiments confirm the theoretical result concerning the exponential convergence of spectral methods for analytic functions. In particular, it is demonstrated that the logarithm of the approximation error decreases linearly as the degree of the approximating polynomial increases. Furthermore, the influence of computational precision on the observed convergence rate is examined.

Keywords: initial value problems; pseudo spectral collocation method; Chebyshev polynomials; Gauss–Lobatto sets; numerical stability.

MSC numbers: 65L60.

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