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Fast and correct computation of spectral representations of functions on Chebyshev grids

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Abstract. The Chebyshev collocation method (CCM) has been successfully and actively used for many years in the numerical solution of differential and integral equations, primarily linear ones. The purpose of this paper is to demonstrate the particularly convenient algebraic structure of the Chebyshev collocation method, which is clearly evident when applied to function interpolation. This article explores several different approaches to using special interpolation grids and the properties of discrete orthogonality with respect to differently preconditioned Chebyshev matrices whose Gram matrix is diagonal. Each of the proposed methods enables the robust calculation of the coefficients of the expansion in Chebyshev polynomials of both the first and second kind for the functions under study. In essence, multiplying the transposed matrix by the vector of function values at grid points yields the coefficients of the expansion of this function in Chebyshev polynomials. Using the Clenshaw method reduces the complexity of this operation to that of the discrete cosine transform (Fourier) for an arbitrary number of interpolation points. Using this approach allows us to dramatically simplify calculations when solving differential and integral equations.
Obtaining the coefficients of the derivative expansion is reduced to multiplying the vector of interpolation coefficients by an upper-triangular differentiation matrix. The coefficients of the antiderivative are obtained by simply multiplying the bidiagonal integration matrix by the vector of interpolation coefficients of the function. Thus, the Chebyshev interpolation method demonstrates the highest efficiency of the collocation method.

Keywords: Interpolation, Chebyshev Polynomials, Collocation Methods, Grids of First and Second Kind
MSC numbers: 83C55, 83C57

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