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A multi-stage Chebyshev collocation method for an approximate solution of a first-order ordinary differential equation

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Abstract. A classical pseudospectral collocation method based on the expansion of the solution in a basis of Chebyshev polynomials is considered. This new approach to forming systems of linear algebraic equations for solving ordinary differential equations with variable coefficients and initial (and/or boundary) conditions allows for a significant simplification of the matrix structure, reducing it to a diagonal form. The solution is a two-step process. The basic algorithm is solving the problem of reconstructing a function from its known derivative and initial/boundary conditions. Reconstructing the expansion coefficients by the basic algorithm amounts to multiplying the transposed matrix of Chebyshev polynomial values on the selected collocation grid by the vector of function values describing the given derivative at the collocation points. Subsequently, multiplying the bidiagonal spectral “inverse” (with respect to the Chebyshev differentiation matrix) matrix by the resulting vector of interpolation coefficients yields all the expansion coefficients of the desired solution except the first one. This first coefficient is determined in the second stage based on the given initial (and/or boundary) condition. The novelty of this approach consists of first identifying a class of functions satisfying the differential equation. Only then solutions from this set that correspond to the given initial conditions are selected. The second stage is the calculation of the integrating factor of the ODE, based on the same basic algorithm, which allows us to reduce the solution of the general equation to an intermediate solution to the derivative recovery problem. To find the final solution, we again use the basic algorithm.

Keywords: initial value problems; pseudo spectral collocation method; Chebyshev polynomials; Gauss–Lobatto sets; numerical stability
MSC numbers: 65L60

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