



On finding the integrating factor of a second-order ordinary differential equation in symbolic form

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Abstract. The problem of integrating a second-order ordinary differential equation $f(x, y, y')y'' + g(x, y, y') = 0$ is considered in symbolic form. First, it is shown how to determine in a finite number of steps whether there exists a function $u(x, y, y')$ such that $f(x, y, y')y'' + g(x, y, y') = du$, and how to find such a function. Following Cheb-Terrab's proposal and by analogy with the case of first-order equations, such equations are called exact. Then, the issue of finding an integrating factor in a finite number of steps is considered, i.e., finding a function μ such that the equation $\mu \cdot (f(x, y, y')y'' + g(x, y, y')) = 0$ is exact. As in the case of first-order equations, the factor can be calculated explicitly under the assumption that it depends on one of the three variables x, y or y' . The case $\mu(y')$ is considered in detail.

Keywords: ordinary differential equations, symbolic integration, integrating multiplier

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