http://mmg.tversu.ru ISSN 2311-1275

Mathematical Modelling and Geometry

Volume 14, No 1, p. 9 – 15 (2026) **Full text**

Darboux Ideals of Hamiltonian Systems

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Abstract. For Hamiltonian systems, a generalization of the concept of Darboux polynomial is proposed. By analogy with the differential ideal, the concept of the Darboux ideal was introduced. The basic properties of such ideals are indicated. The principal Darboux ideal is generated by a Darboux polynomial. For the Darboux ideal of the form $\langle f, g\rangle$, an analog of Jacobi–Poisson theorem is proved. If $J = \langle f, g\rangle$ is a Darboux ideal, then the ideal $\langle f, g, [f, g]\rangle$ is also a Darboux ideal of this ring. Here $[.,.]$ are designated Poisson brackets.

Keywords: Darboux polynomials, ideal, Poisson brackets.

PACS numbers: 07.05.Tp, 02.60.Pn, 02.70.Bf.

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