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Application of Gröbner bases for finding accurate states of equilibrium of migration and population models

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Abstract. The paper considers the using Gröbner basis to accurately find all equilibrium states of migration and population models with competition, as a solution for nonlinear equation system. Example for four-dimensional and six-dimensional models presented. Using a simple example of such a system, it is shown that standard numerical methods find equilibrium states with low accuracy. The proposed approach using the GINV library (Python) and the SageMath computer algebra system makes it possible to reliably find all the roots and obtain them with arbitrary accuracy. The program code and calculation results are provided.

Keywords: Grobner basis, migration-population model, state of equilibrium, nonlinear equation systems.

MSC numbers: 34A34, 13P10.

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