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Using helical flows to test difference schemes for Navier-Stokes equations

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Abstract. Currently, a number of difference schemes are known that approximate the Navier-Stokes equations. Difference schemes are typically tested on two-dimensional problems, such as calculating the Karman street. However, three-dimensional flows present the greatest challenge. In the paper by V. P. Kovalev et al. (2017), two particular solutions to the Navier-Stokes equations were analytically described: the ABC solution and the Gromeka-Beltrami solutions. Both of these solutions describe helical flows, that is, flows in which the velocity curl curl $\vec{v}$ is proportional to the velocity $\vec{v}$. These flows provide a natural test case for comparing the performance of different difference schemes. Here we present the results of testing the schemes from the paper by V. P. Gerdt et al. (2020) on these helical flows.

Keywords: Navier-Stokes equations, finite differences, Gromeka-Beltrami solutions
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