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# On the solution of the Helmholtz equation in an ellipsoid of revolution with a singularity at the focus

**Oleg K. Kroytor**

Patrice Lumumba Peoples' Friendship University of Russia, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation

**e-mail:** kroytor_ok@pfur.ru

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**Abstract.** The propagation of a monochromatic spherical wave emitted from one of the foci of an ellipsoid and reflected at its boundary is considered. From the point of view of geometric optics, the second focus of the ellipsoid should be a singularity of the solution, since the wave reflected from the ellipsoid boundaries "converges" there. In wave optics, on the contrary, no singularity can exist there. The corresponding boundary value problem is formulated within the framework of wave optics and solved using the finite element method. Computer experiments were performed in FreeFem++, using a weak formulation of the problem in a spherical coordinate system. A series of computer experiments demonstrate the changes in this wave with increasing frequency. It is shown that, starting from a certain frequency, a clearly expressed extremum appears in the region of the second focus.

**Keywords:** finite elements, boundary value problems, focus

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