http://mmg.tversu.ru ISSN 2311-1275

Mathematical Modelling and Geometry

Volume 12, No 3, pp. 1 – 7 (2024) **Full text**

Gauss's lemma and integration of a linear ordinary differential equation

Mikhail D. Malykh [1,2,a]

[1] Patrice Lumumba Peoples' Friendship University of Russia, 6 Miklukho-Maklaya St, Moscow, 117198, Russian Federation

[2] Joint Institute for Nuclear Research, 6 Joliot-Curie St. Dubna, Moscow Region, 141980, Russian Federation

e-mail: [a] malykh-md@rudn.ru

Received 28 November 2024. Published 01 December 2024.

Abstract. The problem of factorization of second-order linear ordinary differential equations (LODEs) is considered. Over a polynomial ring $k[x]$, the operator $L = aD^2 + bD + c$ is considered, where D is the differentiation operator with respect to x, and the coefficients a, b, c are polynomials from the ring $k[x]$. A Gaussian integral factor is introduced as a polynomial μ such that μL can be factorized into linear factors with coefficients from the ring $k[x]$. If the operator L can be factorized over the field of quotients of the ring $k[x]$, then a Gaussian factor exists. However, in contrast to the classical case of factorization in the ring $\mathbb{Q}[x]$, it is impossible to prove a complete analogue of the Gauss lemma, that is, to assume that $\mu = 1$. A possible generalization of Gauss's lemma to this case is presented, and from it a connection is derived between the Gaussian multiplier and finite-zero solutions of linear ordinary differential equations (LODEs).

Keywords: linear ordinary differential equations, symbolic integration, integrating multiplier

PACS numbers: 07.05.Tp, 02.60.Pn, 02.70.Bf

References

[1] Fogaça Matheus Janczkowski, Cardoso Eduardo Lenz. A systematic approach to obtain the analytical solution for linear second order ordinary differential equations: part I // Journal of the Brazilian Society of Mechanical Sciences and Engineering. 2024. Vol. 46, no. 214.

[2] Demidov S. S. On the history of the theory of linear differential equations // Istoriko-matematicheskie issledovaniya. 1985. Vol. 28. P. 78–97.

[3] Berkovich L. M. Factorization of ordinary linear differential operators that can be transformed into operators with constant coefficients //Izv. Vyssh. Uchebn. Zaved. Mat. 1965. no. 4. P. 8–16.

[4] Berkovich L. M. Factorization of ordinary linear differential operators that can be transformed into operators with constant coefficients. II //Izv. Vyssh. Uchebn. Zaved. Mat. 1967. no. 12. P. 3–14.

[5] Berkovich L. M. Factorization and transformations of differential equations. D&C, 2002.

[6] Moses J. Symbolic Integration: The Stormy Decade // Communications of the ACM. 1971. Vol. 14, no. 8. P. 548–560.

[7] Singer M.F. Liouvillian first integral of differential equations // Trans. Amer. Math. Soc. 1992. Vol. 333, no. 2. P. 673–688.

[8] Malykh M. D. On integration in quadratures of the first-order ordinary differential equations //Mathematical Modelling and Geometry. 2025. Vol. 13, no. 2. P. 16–31.

[9] Cheb-Terrab E. S., Roche A. D. Integrating factors for second order ODEs //Journal of Symbolic Computation. 1999. Vol. 27, no. 5. P. 501–519.

[10] Lang S. Algebra. Springer, 2002.