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A note on Hamiltonian simulation in the Pauli basis

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Abstract. In this note, we consider a multiqubit quantum system with a Hamiltonian which can be decomposed into a few parts in such a way that each of these parts is a finite linear combination of pairwise anticommuting Pauli operators. It turns out that such a decomposition allows one to enhance both quantum and classical algorithms for Hamiltonian simulation, at least for Hamiltonians that are sparse in the Pauli basis. The advantage is achieved due to the possibility to represent in a simple closed form the exponential of a linear combination of pairwise anticommuting Pauli operators. We consider two illustrative models, which demonstrate the quantum computational advantage for the Trotter-Suzuki algorithm. We also show how an anticommutatively decomposed Hamiltonian can be efficiently simulated on a classical computer.

Keywords: Hamiltonian simulation, anticommuting operators, n-qubit quantum system, Pauli basis, operator exponential
MSC numbers: 81-08, 81R05

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