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## **Dynamic analysis of a fractional-order delayed SIQR epidemic model for COVID-19 pandemic**

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**Abstract.** In this paper, we build a fractional-order delayed SIQR epidemic model for COVID-19 pandemic. By leveraging the linearization method and the Laplace transform, the characteristic equation of the linearized system corresponding to the model is derived. The relationship between time delay and the stability of the positive equilibrium point is then explored through the examination of the roots of the characteristic equation and the cross-sectional conditions. Furthermore, a formula for calculating the critical value of time delay for the Hopf bifurcation is derived. Finally, suitable system parameters are chosen for numerical simulation to validate the soundness of the theoretical analysis.

**Keywords:** Fractional calculus; COVID-19 model; Stability; Hopf bifurcation; Delay

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