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Thin-film waveguide Lüneburg lens in the model of adiabatic guided modes

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Abstract. A mathematical model is presented that describes the processes of propagation and transformation of coherent electromagnetic radiation in a multilayer three-dimensional (3D) smoothly irregular integrated optical waveguide, called the model of adiabatic guided modes. Its presentation and individual applications in smoothly irregular integrated optical waveguides contain two short stories:

- two-dimensional evolution of guided modes is described;
- boundary conditions are formed on non-horizontal planes tangent to media interfaces, which lead to the description of hybridization of guided modes and other interesting phenomena.

The model of adiabatic guided modes generalizes the cross-section method (reference waveguide method) with nonlocal boundary conditions for the transverse guided mode operator in the reference waveguide cross section to the case of two-dimensional evolution, leading to the description of a number of new effects.

Keywords: mathematical model, 2D and 3D geometry, generalized Luneburg lens, vector Maxwell equations, nonlocal boundary conditions, asymptotic method, adiabatic guided modes, quasi-TE and quasi-TM modes, electromagnetic radiation, waveguide optoelectronics, eigenvalues, eigenfunctions, Nelder-Mead simplex method, numerical integration, Cauchy problem, system of ordinary differential equations, system of linear algebraic equations, Tikhonov regularization method

MSC numbers: 35Pxx, 35Qxx, 39-08, 65Hxx, 65Nxx, 78-04, 78-10

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