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Equilibrium spherical shell of condensed matter around a scalar naked singularity

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Abstract. From a observational point of view, the central regions of scalar naked singularities possess a very special spacetime geometry, which in turn leads to existence of unusual orbital motion near the centers. We consider naked singularities in the model of static, spherically symmetric, asymptotically flat configurations of a self-gravitating scalar field minimally coupled to gravity. In this case, the effective potential of test particles orbiting around a scalar naked singularity has a minimum even at zero specific angular momentum. This means that the baryonic matter, captured by the naked singularity in the region of gravitational potential well near the center, will eventually be concentrated in some spherical shell, which will be in hydrostatic equilibrium after cooling. We find the conditions of hydrostatic equilibrium of the shell for the polytropic equation of state. In order to show the observational consequences of the possible existence of such configurations as real astrophysical objects, in particular in relation to tidal disruption events, we consider a simple illustrative example.

Keywords: scalar naked singularity, orbital motion, polytropic equation of state, potential well
MSC numbers: 83C10, 83C57

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