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Equilibrium spherical shell of condensed matter around a scalar naked singularity

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Abstract. From a observational point of view, the central regions of scalar naked singularities possess a very special spacetime geometry, which in turn leads to existence of unusual orbital motion near the centers. We consider naked singularities in the model of static, spherically symmetric, asymptotically flat configurations of a self-gravitating scalar field minimally coupled to gravity. In this case, the effective potential of test particles orbiting around a scalar naked singularity has a minimum even at zero specific angular momentum. This means that the baryonic matter, captured by the naked singularity in the region of gravitational potential well near the center, will eventually be concentrated in some spherical shell, which will be in hydrostatic equilibrium after cooling. We find the conditions of hydrostatic equilibrium of the shell for the polytropic equation of state. In order to show the observational consequences of the possible existence of such configurations as real astrophysical objects, in particular in relation to tidal disruption events, we consider a simple illustrative example.

Keywords: scalar naked singularity, orbital motion, polytropic equation of state, potential well

MSC numbers: 83C10, 83C57

1 Introduction

At present, we still do not exactly understand the nature of strongly gravitating objects in the centers of normal galaxies. It is currently believed that supermassive black holes are the most likely candidates for the role of these objects. However, other possibilities that are considered in the literature [1] include, in particular, wormholes [2, 3, 4], naked singularities [5, 6, 7] or a dense core with the same outer geometry [8], and boson stars [9, 10]. The best observational results have been obtained for the centres of our Galaxy and of M87 [11, 12, 13, 14, 15, 16], and they do not allow us to unambiguously state that we are dealing with black holes. Moreover, it is shown in [6] using a simple model that a naked singularity can have both a shadow and a photon sphere. In fact, observations of the orbits of stars near the centres have a key role in dealing with this question, but there are some obvious problems with the reasonable geometrical interpretation of such observations.

The observational efficiency directly depends on a model in which the astrophysical data for the central objects will be interpreted. In this article we consider static, self-gravitating, spherically symmetric configurations that are formed and supported in equilibrium by a self-interacting real scalar field minimally coupled to gravity. Our motivation for exploring properties of such mathematical model arose from the common belief that one should not think of the central objects in galaxies as being in vacuum, because dark matter surrounding the centers of galaxies cannot be ignored. In our approach, a nonlinear scalar field represents a kind of anisotropic fluid and, in fact, is introduced as a model of dark matter [17, 18, 19]; note that it does not matter whether scalar fields exist in nature.

The geometry of spacetime and the orbital motion around scalar black holes have been studied much better than the same characteristics of scalar naked singularities. The main purpose of this article is to study one of the key features of scalar naked singularities, namely, the presence of a minimum of the lapse metric function near the center, while in the outer region, the geometry of spacetime is asymptotically flat and only slightly differs from the Schwarzschild spacetime. Note also that the family of Schwarzschild naked singularities is parameterized by negative masses and do exhibit only gravitational repulsion everywhere.

The article is organised as follows. In Sect. 2 we describe the necessary mathematical background for static, spherically symmetric scalar field configurations restricting our attention to the case of the minimal coupling between curvature and a real scalar field. Using the so called method of restored potential (or, in other terminology, the so-called inverse problem method for self-gravitating spherically symmetric scalar fields), we present a simple example which illustrates the main characteristic features of scalar naked singularities. Sect. 3 is devoted to studying bound orbits of free neutral massive test particles; the main astrophysical objects that we have in mind are the bound orbits of stars in the centres of galaxies. We show that there is a degenerated stable circular orbit in which test particles have zero angular momentum and remains at rest all the time. In what follows, we will refer to this orbit as *orbit of rest*. In Sect. 4 we study the equilibrium of a spherical shell of condensed baryonic matter captured by the scalar naked singularity, which is considered in Sect. 2. In doing so, we restrict our attention on the polytropic equation of state and qualitatively consider a possible explanation of bright flares (the so-called tidal disruption events) in the centers of galaxies.

In this article, we use the geometrical system of units with $G = c = 1$. In tensor notation, we use the summation convention over repeated indices, and Greek indices take the values $0, 1, 2, 3$. We also adopt the metric signature $(+\ -\ --)$ and the definitions $R^i_{jkl} = \partial_k \Gamma^i_{jl} - \ldots$, $R_{jl} = R^i_{jil}$ for the curvature and the Ricci tensor, respectively.

2 Scalar naked singularities

The action with the minimal coupling between curvature and a real scalar field ϕ has the form

$$\Sigma = \int \left(-\frac{1}{16\pi} S + \frac{1}{8\pi} \left(\langle d\phi, d\phi \rangle - 2V(\phi) \right) + \mathcal{L}_m \right) \sqrt{|g|}\, d^4x \,, \tag{1}$$

where $V(\phi)$ is a self-interaction potential, the angle brackets denote the scalar product with respect to the spacetime metric, and $\mathcal{L}_m$ is the matter Lagrangian. We will write the metric of a spherically symmetric spacetime in the Schwarzschild-like coordinates as

$$ds^2 = Adt^2 - \frac{dr^2}{f} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \qquad A = f e^{2F}, \tag{2}$$

where the metric functions A, F, and f depend only on the radial coordinate r. These functions should be the result of solving the Einstein field equations

$$\mathcal{R}_{ij} - \frac{1}{2} S g_{ij} = 8\pi \mathcal{T}^{\phi} + 8\pi \mathcal{T}^{(m)}.$$

Assuming that the matter in the action (1) is represented by an isentropic perfect fluid with polytropic equation of state

$$P = K\rho^{1+1/n}, \tag{3}$$

and the energy-momentum tensor

$$\mathcal{T}^{(m)}_{ij} = (\rho + P) u_i u_j - P g_{ij},$$

one can use the standard variational procedure for the scalar field in order to find the four well-known independent Einstein-Klein-Gordon-fluid matter equations. These equations, together with the equation of state (3), make up the complete system of field equations for the configuration under consideration and have the form (a prime means differentiation with respect to r)

$$-\frac{f'}{r}-\frac{f-1}{r^2}=\phi'^2 f+2V+\frac{q^2}{r^4}+8\pi\rho\,, \tag{4}$$

$$\frac{f}{r}\left(2F'+\frac{f'}{f}\right)+\frac{f-1}{r^2}=\phi'^2 f-2V-\frac{q^2}{r^4}+8\pi P\,, \tag{5}$$

$$-\,f\phi''-\frac{\phi'}{2}f'-\phi' f\left(F'+\frac{1}{2}\frac{f'}{f}+\frac{2}{r}\right)+\frac{dV}{d\phi}=0\,, \tag{6}$$

$$P'\,+\,(\rho+P)\left(\frac{1}{2}\frac{f'}{f}\,+\,F'\right)\,=\,0\,. \tag{7}$$

Note first that eq. (7) can be written in the form

$$P'\,+\,\frac{1}{2}\frac{A'}{A}(\rho+P)\,=\,0\,. \tag{8}$$

Second, by adding eqs. (4) and (5), one can reduce (5) to the form

$$F'=r\phi'^2+\frac{4\pi r}{f}\,(\rho+P)\,. \tag{9}$$

In what follows, we use the background approximation, assuming that the mass of condensed baryonic matter is relatively small, so that the spacetime metric is determined only by a self-gravitating scalar field. In other words, we can eliminate the pressure and density of matter in the right hand sides of eqs. (4) and (9).

The form of the self-interaction potential $V(\phi)$ is unknown a priori, but the method of restored potential is suitable for all physically admissible potentials at once. This method was proposed in [20, 21, 22] and later was explored in [23, 24, 25, 26, 27] and applied, for example, in [28, 29, 30]. In this article we use the following quadrature formulae [26]:

$$F(r)=-\int_r^\infty \phi'^2 r\,dr\,,\quad \xi(r)=r+\int_r^\infty\left(1-\mathrm{e}^F\right)dr\,, \tag{10}$$

$$A(r)=2r^2\int_r^\infty\frac{\xi-3M}{r^4}\,\mathrm{e}^F\,dr\,,\quad f(r)=\mathrm{e}^{-2F}A\,, \tag{11}$$

$$\widetilde{V}(r)=\frac{1}{2r^2}\left(1-3f+r^2\phi'^2 f+2\,\mathrm{e}^{-F}\,\frac{\xi-3M}{r}\right), \tag{12}$$

where $\widetilde{V}(r)=V(\phi(r)$ and it is required that one of the function ϕ, F or ξ is given. Thus, the quadratures (10) and (11) determine the metric functions, while the formula (12) determines the self-interaction potential as a function of the radial coordinate. Next, assuming that $\phi(r)$ is piecewise monotone, one can restore the potential $V(\phi)$. It can be seen directly from the quadratures that for all $r > 0$

$$F \leq 0, \quad e^F \leq 1, \quad \xi > 0, \quad \xi' = e^F > 0, \quad \xi'' = r\phi'^2 e^F \geq 0. \tag{13}$$

The boundary conditions for ξ are

$$\xi = \xi(0) + \alpha r + O\big(r^2\big) \ (0 \leq \alpha \leq 1), \ \ r \to 0, \qquad \xi = r + o(1), \ \ r \to \infty, \tag{14}$$

and $\xi(0) > 0$ if ϕ' is not identically zero.

We start with the function

$$\xi(r) = \sqrt{r^2 + ar + a^2} - \frac{a}{2}\,, \tag{15}$$

where $a > 0$. Next, by direct differentiation, we obtain

$$\xi'(r) = e^{F(r)} = \frac{a + 2r}{2\sqrt{r^2 + ar + a^2}}\,,$$

$$\phi'(r) = \sqrt{\xi''(r)e^{-F(r)}/r} = \frac{a}{\sqrt{(2r/3)(a + 2r)(r^2 + ar + a^2)}}\,.$$

We do not need an explicit expression for the field function (in terms of the incomplete elliptic integral of the first kind), and we note only that it decreases at infinity as r^{-1} and diverges logarithmically at $r = 0$.

The metric function A can be found in analytical form by direct integration in (11):

$$A(r) = 1 + \frac{a}{3r} - \sqrt{r^2 + ar + a^2}\left(1 + \frac{7r}{4a} - \frac{37r^2}{8a^2}\right)\frac{a + 6M}{6ar} - 3\ln\left[5 + \frac{8a}{r} + \frac{8a^2}{r^2} + \left(4 + \frac{8a}{r}\right)\left(1 + \frac{a}{r} + \frac{a^2}{r^2}\right)^{1/2}\right]\frac{(a + 6M)r^2}{64a^3} + \left(3\ln 3 - \frac{74}{3}\right)\frac{(a + 6M)r^2}{32a^3}\,. \tag{16}$$

At infinity, the metric behaves like the Schwarzschild solution. In particular,

$$A(r) = 1 - \frac{2M}{r} + \frac{3a^3 + 18Ma^2}{40r^3} + O(r^{-4})\,, \ \ r \to \infty\,, \tag{17}$$

that is, M is the Schwarzschild mass for this configuration. Near the center

$$A(r) = \frac{a - 6M}{6r} + \frac{5a - 18M}{8a} + + \frac{9a + 54M}{16a^2} r + O(r^2)\,, \ \ r \to 0, \tag{18}$$

so that the configuration is a naked singularity if $a > 6M$. The interval $a \in [0, 6M)$ corresponds to black holes (the value $a = 0$ does to the Schwarzschild black hole of mass M). The value $a = 6M$ determines a regular configuration, but in this case case fine tuning of the parameters is required.

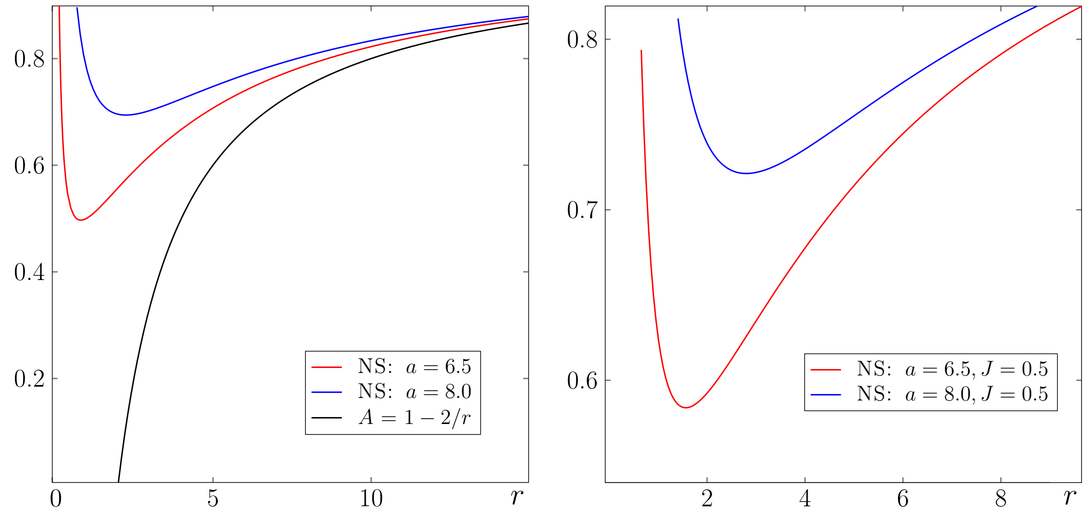

Figure 1: On the left: the metric functions $A(r)$ (16) for various values of the parameter a; the Schwarzschild solution $A = 1 - 2/r$ with the same mass ($M = 1$). On the right: the effective potentials (20) of freely falling massive particles.

3 Bound orbits

In what follows, it is convenient to choose the unit of length by setting $M = 1$.

In any static, spherically symmetric spacetime a massive test particle has three integrals of motion. For the metric of the form (2) they are

$$(19)\frac{dt}{ds} = \frac{E}{A}\,, \quad \frac{d\varphi}{ds} = \frac{J}{r^2}\,, \quad \left(\frac{dr}{ds}\right)^2 = \mathrm{e}^{-2F}\big(E^2 - V_{eff}\big)\,, \tag{19}$$

$$V_{eff} = A\left(1 + \frac{J^2}{r^2}\right), \tag{20}$$

where V_{eff} is the effective potential, E is the specific energy, and J is the specific angular momentum of a massive test particle. Fig. 1 shows the typical behavior of the metric function $A = A(a, r)$ and the effective potential $V_{eff} = V_{eff}(a, J, r)$ for various values of the parameter a and specific angular momentum J.

Circular orbits or, more generally, bound orbits exist if the effective potential has a minimum. For black holes of any origin, the effective potential reaches a minimum only at certain values of the specific angular momentum, $J > J_{ISCO}$, where J_{ISCO} determines the innermost stable circular orbit. The key feature of scalar field naked singularities is the existence of some radius r_{min} at which the metric function $A(a, r) = V_{eff}(a, 0, r)$ reaches a minimum value. This means, in turn, that a test particle with zero angular momentum and the energy $E_{min} = V_{eff}(a, 0, r_{min})$ will remain in the state of rest all the time. Particles with zero angular momentum and

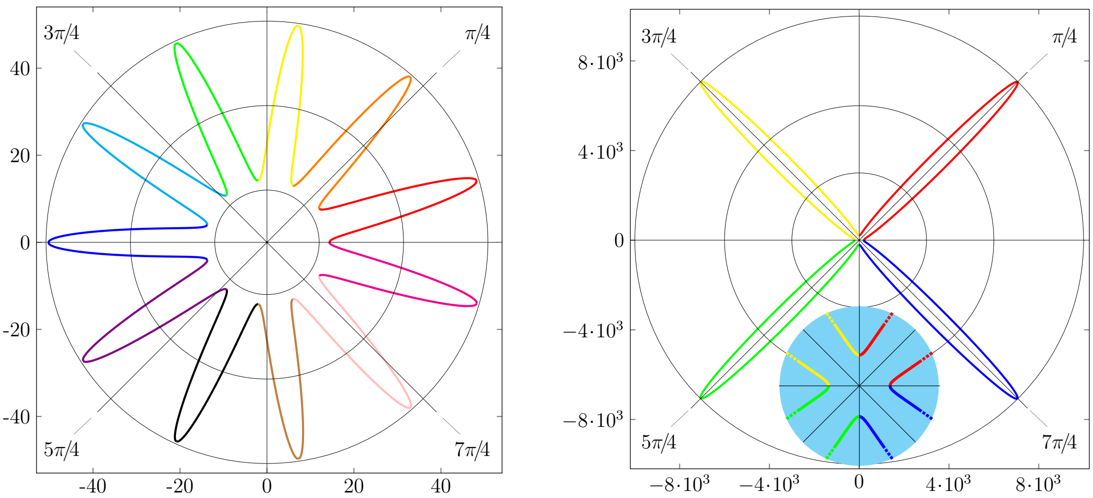

Figure 2: Left panel: The shape of the orbit that could be formed by a realistic astrophysical configuration in the process of capturing a star; the parameters are $a = 10.0$, $J = 1.1$, $E = 0.9606$, $r_{min} = 18.2$, $r_{max} = 50.4$. Right panel: an extremely prolate orbit; the parameters are $a = 10.0$, $J = 4.4$, $E = 0.9865$, $r_{min} = 5.43$, $r_{max} = 8.3 \cdot 10^3$.

energy $E_{min} < E < 1$ will perform radial oscillations. If a particle has small nonzero angular momentum, then its orbit can have very unusual shape (see Fig. 2). The shape of a bound orbit is determined by the equation

$$\varphi'(E, J, r) = 2J \, \frac{\mathrm{e}^F}{r^2\sqrt{E^2 - V_{eff}}} \,,$$

which follows from (19) and can be easily solved numerically. Note that the parameters a and J in Fig. 2 were specially chosen in order to obtain closed orbits, but generally speaking this is not the case.

Taking into account these features of the orbital motion near the center of a scalar naked singularity, the following question arises: what is the late-time asymptotic behavior of matter captured by the naked singularity from its outer region. We cannot expect to solve this problem rigorously, even if we assume that the initial conditions are known. However, the examples discussed above show that the movement of matter in the central region of the potential well should be highly chaotic and collision-dominated. Thus, these examples partly justify our intuitive assumption that the matter is cooling down and, after some time, comes to the equilibrium state of an isentropic fluid. In the next section, we study the parameters of the equilibrium state of a spherical shell of condensed matter in the gravitational potential well of a scalar naked singularity.

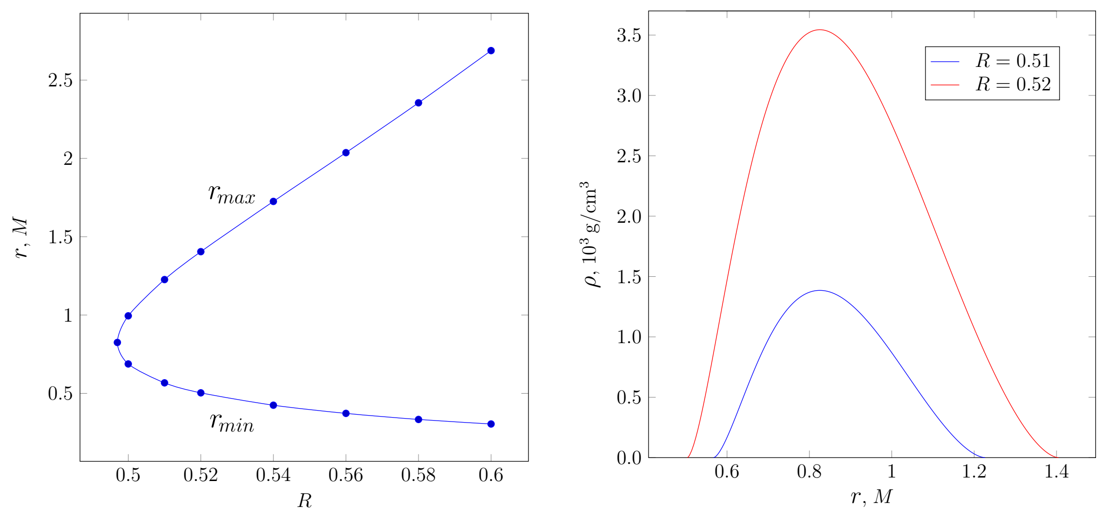

Figure 3: The plot on the left shows r_{min} and r_{max} (the marginal radii of the spherical shell of condensed baryonic matter) as functions of the "constant thickness" R. The plot on the right shows typical density distributions in the shell.

4 Equilibrium spherical shell of condensed baryonic matter with the polytropic equation of state

In this section, it is convenient to go back for a while to the usual *length-mass-time* units, for example to Gauss units. Recall that we neglected the contribution of matter to the geometry of spacetime near the centers of scalar naked singularities. Let us write out once again the polytropic equation of state (3) and the hydrostatic equilibrium equation (8),

$$P = K\rho^{1+1/n}, \qquad P' + \frac{1}{2}\frac{A'}{A}(\rho c^2 + P) = 0\,, \tag{21}$$

since it is they that determine the properties of the spherical shell of condensed matter in the background approximation. It turns out that the general solution of this set of equations can be found using the Emden substitution

$$\rho = \rho_0\theta^n, \quad 0 \le \theta(r) \le 1\,, \tag{22}$$

where the value $\theta = 0$ corresponds to the inner and outer edges of the shell, while the value $\theta = 1$ with $\theta' = 0$ determines the maximum of $\rho(r)$. As a result we have

$$P = K\rho_0^{1+1/n}\theta^{n+1}, \quad P' = (n+1)K\rho_0^{1+1/n}\theta^n\theta',$$

$$\rho c^2 + P = \rho_0\theta^n c^2 + K\rho_0^{1+1/n}\theta^{n+1} = \rho_0\theta^n\,(c^2 + K\rho_0^{1/n}\theta)\,,$$

so that the second equation in (21) takes the form

$$(n+1)K\rho_0^{1/n}\theta' + \frac{1}{2}\frac{A'}{A}(c^2 + K\rho_0^{1/n}\theta) = 0\,.$$

From this equation and (23), we obtain the general solution for equilibrium shell of condensed matter in the background approximation:

$$\rho = \left(\frac{c^2}{K}\right)^n \left[\left(\frac{R}{A}\right)^{\frac{1}{2(n+1)}} - 1\right]^n . \tag{23}$$

The dimensionless constant R determines (or is determined by) the inner and outer edges as the two roots, say r_{min} and r_{max}, of the equation $A(r) = R$; note that $R > A(r_c)$, where $A'(r_c) = 0$ and r_c is the radius of the sphere with the highest density. The factor K in (21) has the dimension $[K] = \mathrm{cm}^{3/n+2}/(\mathrm{g}^{1/n}\mathrm{s}^2)$ and depends only on the index n and the chemical composition, temperature, and specific entropy of matter. We take reasonable values $n = 5/3$ for the polytropic index and $\rho_* = 10^7\,\mathrm{g/cm}^3$ for the characteristic density $\rho_* = (c^2/K)^{5/3}$. In our model the metric function A is given by the expression (16) and has

$$r_c \approx 0.825\,, \qquad A(r_c) \approx 0.497\,. \tag{24}$$

The main features of the model are presented in Fig. 3. Let $m = m(R)$ be the mass of baryonic matter contained in the spherical shell. We will also assume $M = 4\cdot 10^6 M_\odot$ for the central mass, since this value corresponds to the typical masses of galactic centers including Sgr A*. Then for the values R taken on the right panel in Fig. 3, we find $m(0.51) \approx 2.3\cdot 10^3 M_\odot$ and $m(0.52) \approx 7.1\cdot 10^3 M_\odot$.

5 Conclusion

The results of this article are threefold. First, we obtain the full system of the spherically symmetric, static Einstein-Klein-Gordon-fluid matter equations with the polytropic equation of state. In the background approximation, that is, neglecting the contribution of matter, we have constructed a two-parameter family of asymptotically flat configurations of a self-gravitating scalar field minimally coupled to gravity; in some domain of parameters, these configurations are naked singularities. Second, bound orbits near the centres of scalar naked singularities are studied. It is shown that the geodesic motion of test particles in this region is very chaotic, so that they will be over time concentrated in the gravitational potential well of a naked singularity. This assumption directly leads to the model of a spherically symmetric shell of baryonic condensed matter in a static equilibrium. And third, using the Emden substitution, we find the general solution of the hydrostatic equilibrium equation in the background approximation. In our model we have in mind that test particles are stars and gas clouds in the centres of galaxies, the nonlinear scalar field is considered as an idealized model of dark matter, and the shell of condensed matter is formed by capturing baryonic matter from outer region; in other words, we assume that scalar naked singularities may exist in the centers of galaxies. In particular, the shells of condensed matter give us an alternative explanation of the so-called tidal disruption events [31, 32, 33]. Namely, these events may be the result of collisions of stars with a shell of condensed matter. In this connection, we have considered the density distribution in an illustrative example with more or less realistic parameters. However, this model can become really popular when the resolution of observations reaches a value of several tens of masses (in units of length) of central objects.

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