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Slowly rotating spacetimes with scalar hair

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Abstract. We consider asymptotically flat, rotating configurations of a self-gravitating real scalar field minimally coupled to Einstein gravity. For the sake of completeness and to introduce convenient notation, we formulate the Einstein-Klein-Gordon equations in the orthonormal tetrad uniquely associated with the metric of a nonstationary axisymmetric spacetime. First, in the stationary case, we study Bianchi identities to find appropriate variants for reducing the Einstein-Klein-Gordon system to a complete subsystem of independent equations. It turns out that this procedure can be performed in three ways. Second, assuming that the rotation is slow, the full system of these equations is linearized about a spherically symmetric scalar field configuration, which may be thought of as some exact solution. In doing so, we do not assume that the scalar field is small, and do not specify the type of the basic spherically symmetric configuration, which can be a black hole, a naked singularity, or a wormhole.

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