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# Covariant series in the normal neighborhood of a submanifold

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**Abstract.** We consider the covariant series in a some normal neighborhood of a submanifold. Such a neighborhood is a generalization of the normal neighborhood of a point. We discuss how the coefficients of the covariant Taylor series of an arbitrary tensor field can be expressed in terms covariant derivatives of the torsion, Riemann curvature and the field under consideration. We also discuss the algorithm of calculating coefficients of a pseudo-Riemannian metric with respect to the corresponding metric connection without torsion. As an example, we calculate the covariant expansion of the Schwarzschild metric in the normal tubular neighborhood of a circular orbit up to the fifth order using the Fermi coordinate system.

**Keywords:** Riemann manifold, normal neighborhood, covariant series, pseudo-Riemannian metric

**MSC numbers:** 53A45, 53Bxx, 83-08, 83Cxx

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