



Particle configurations in the $NN\bar{K}$ system

Igor Filikhin^{1,a}, Yury B. Kuzmichev² and Branislav Vlahovic¹

¹ Department of Mathematics&Physics, North Carolina Central University, Durham, NC 27707, USA

² Yaroslavl State Pedagogical University, 150000, Yaroslavl, Russia

e-mail: ^a ifilikhin@nccu.edu

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Abstract. Three-body AAB model for the $NN\bar{K}(s_{NN} = 0)$ kaonic cluster is considered based on the configuration space Faddeev equations. Within a single-channel approach, the difference between masses of nucleons and kaons and the charge independence breaking of nucleon-nucleon interaction are taken into consideration. We define the particle configurations in the system according to the particle masses and pair potentials. There are two sets of the particle configurations, $pp\bar{K}^-$, $np\bar{K}^0$ and $nn\bar{K}^0$, $np\bar{K}^-$, charged and neutral. The three-body calculations are performed by applying NN and $N\bar{K}$ phenomenological isospin-dependent potentials. The mass and energy spectra related to the particle configurations are presented. We evaluate the mass and energy uncertainties for the $NN\bar{K}$ model. An analogy to NNN model for the ${}^3\text{H}$ and ${}^3\text{He}$ nuclei is proposed.

Keywords: mesic nuclei, kaon-baryon interactions, Faddeev equations

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1. Introduction

The quasi-bound state of the kaonic cluster " ppK^- " [1] has been intensively debated during the last time. The theoretical predictions for the binding energy for this $NN\bar{K}(s_{NN} = 0)$ system obtained within different models demonstrated significant disagreement with the values derived from early experimental data [2]. The experimentally motivated value was given to be about -100 MeV (deeply bound state) [3, 4]. The discussions about the experimental background and theoretical interpretations can be found in Refs. [5, 6, 7, 8, 9, 10].

The newest J-PARC E15 experiments [11, 12, 13] had reported the value between -43 and -47 MeV. This value is close to the theoretical prediction made within the phenomenological approach of Ref. [14]. The numerical analysis [15, 16] shows that there is a lower bound of the " ppK^- " energy for the phenomenological potentials having the Gaussian form. The lower bound is about -60 MeV for the $N\bar{K}$ potential proposed in Ref. [14]. This potential has a significant difference for the isospin singlet and triplet channels. The isospin singlet component of the $\bar{K}N$ potential generates a quasi-bound state corresponding to the $\Lambda(1405)$ resonance below the K^-p threshold. The discussed is the nature of the $\bar{K}N$ interaction, which would include two channels $N\bar{K}$ quasi-bound state and $\pi\Sigma$ resonance [14, 19, 20, 17, 18]. To treat this complication, the Akaishi-Yamazaki (AY) $N\bar{K}$ potential [14] takes into account the $\pi\Sigma$ coupling effectively. This effective $\bar{K}N$ interactions have a strong attraction in the singlet $I = 0$ channel and a weak attraction in the triplet $I = 1$ channel. The two-body threshold is close to the bound state energy of $\Lambda(1405)$ (about -27 MeV) as the K^-p bound pair. The binding energy of ppK^- obtained within $NN\bar{K}(s_{NN} = 0)$ isospin model is about -48 MeV [14]. Similar results have been obtained within different models in Refs. [21, 22, 23, 25].

In this work, we present a model study for the " ppK^- " kaonic cluster within the isotopic-spin formalism. Recently, the isospin model for $NN\bar{K}$ system has been presented in the terms of the particle sets. In particular, the particle channels ppK^- and $pn\bar{K}^0$ have been defined in Refs. [26, 27] for the " ppK^- " cluster due to the possible particle transition $n\bar{K}^0 \longleftrightarrow pK^-$. The channels are coupled. Thus, the system may be found as ppK^- or $pn\bar{K}^0$ at the same time with different projections of total isospin ($I=1/2$). Such consideration allows to definite a particle model for the $NN\bar{K}(s_{NN} = 1)$ cluster [28]. This "particle representation" is essentially based on the "charge" isospin basis [29], which can be obtained by unitary rotation from a "natural" isospin basis [30]. Both formulations are equivalent. However, the last one does not support the channel representation. In the isospin formalism, the mesons \bar{K}^0 and K^- are two isospin states of the \bar{K} particle with the isospin of $\frac{1}{2}$. Nucleon is the isospin $\frac{1}{2}$ particle having two states (proton and neutron) with different projections. The total isospin is a sum of the spins of the three particles, which is independent of their other physical properties. From another side, the projections of particle isospin are defined by particle charges. It is the reason to consider "charge basis" instead of isospin "natural" basis [30].

According to Ref. [30], to separate particles sets of the $NN\bar{K}(s_{NN} = 0)$ system,

we take into account the difference between masses of nucleons and kaons and the charge independence breaking of nucleon-nucleon interaction. After that, the $NN\bar{K}(s_{NN} = 0)$ system is represented by four configurations: ppK^- , npK^- , $np\bar{K}^0$, $nn\bar{K}^0$. In the present work, we illustrate this separation and show the results of calculations for the $NN\bar{K}(s_{NN} = 0)$ bound state. The calculated are mass and energy spectra of the system related to the particle configurations.

Our consideration is based on the configuration space Faddeev equations [31]. For the calculations, we apply the phenomenological Akaishi-Yamazaki $N\bar{K}$ potential [14] and nucleon-nucleon potential with charge symmetry breaking correction [32]. The Coulomb potential is ignored in this consideration due to the Coulomb correction for three-body energy may cover the effect considered. Also, we take into account that the Coulomb force violates the AAB symmetry of the npK^- particle configuration.

2. Faddeev equation for $NN\bar{K}$ system

The kaonic cluster $NN\bar{K}$ is represented by the three-body system with two identical particles (AAB system). The total wave function of the AAB system is decomposed into the sum of the Faddeev components U and W corresponding to the $(AA)B$ and $A(AB)$ types of rearrangements: $\Psi = U + W - PW$, where P is the permutation operator for two identical particles. In the expression for Ψ , the sign “-” corresponds to two identical fermions. The wave function is antisymmetrized according to two identical fermions in the system. Each component is expressed by corresponding Jacobi coordinates. For the system the set of the Faddeev equations is presented by two equations for the components U and W [33]:

$$\begin{aligned} (H_0 + V_{AA} - E)U &= -V_{AA}(W - PW), \\ (H_0 + V_{AB} - E)W &= -V_{AB}(U - PW), \end{aligned} \quad (1)$$

where H_0^U and H_0^W are the kinetic energy operators presented in the Jacobi coordinates for corresponding rearrangement. In the presented work, we consider the s -wave approach for the AAB systems. The total angular momentum and angular momenta in the subsystem are equal to zero. In Eq. (1), the Faddeev component U (W) is expressed in terms of spin and isospin spaces: $U = \mathcal{U}\chi_{spin} \oplus \eta_{isospin}$. We consider the $NN\bar{K}$ system with the triplet isospin state of the nucleon pair $I_{NN} = 1$. The isospin basis for $NN\bar{K}$ system in the state $I = 1/2$ and $I_3 = 1/2$ can be written using the isospin functions: $\eta_{+-+} = \eta_+(1)\eta_-(2)\eta_+(3)$, $\eta_{-++} = \eta_-(1)\eta_+(2)\eta_+(3)$, $\eta_{++-} = \eta_+(1)\eta_+(2)\eta_-(3)$. The spin states of the $NN\bar{K}$ system is described by spin states of nucleon pair which is the spin-singlet state.

The separation of spin/isospin variables leads to the following form of the Faddeev equations:

$$\begin{aligned} (H_0^U + v_{NN}^t - E)\mathcal{U} &= -v_{NN}^t(-\frac{1}{2}\mathcal{W}^t - \frac{\sqrt{3}}{2}\mathcal{W}^s - \frac{1}{2}p\mathcal{W}^t - \frac{\sqrt{3}}{2}p\mathcal{W}^s), \\ (H_0^W + v_{\bar{K}N}^s - E)\mathcal{W}^s &= -v_{\bar{K}N}^s((-\frac{1}{2}\mathcal{U} + \frac{\sqrt{3}}{2}p\mathcal{W}^t - \frac{1}{2}p\mathcal{W}^s), \\ (H_0^W + v_{\bar{K}N}^t - E)\mathcal{W}^t &= -v_{\bar{K}N}^t(-\frac{\sqrt{3}}{2}\mathcal{U} + \frac{1}{2}p\mathcal{W}^t + \frac{\sqrt{3}}{2}p\mathcal{W}^s). \end{aligned} \quad (2)$$

Here, the isospin singlet and triplet \mathcal{W} components of the wave function and corresponding isospin components of the potentials are noted by indexes s and t . The exchange operator p acts on the particles' coordinates only. The s -wave spin/isospin dependent V_{NN} and $V_{N\bar{K}}$ potentials are assumed for the corresponding interactions.

3. Particle configurations

The kaonic cluster $NN\bar{K}$ is represented by the three-body system with two identical particles (AAB system). The total wave function of the AAB system is decomposed into the sum of the Faddeev components U and W corresponding to the $(AA)B$ and $A(AB)$ types of rearrangements: $\Psi = U + W - PW$, where P is the permutation operator for two identical particles. In the expression for Ψ , the sign "−" corresponds to two identical fermions. The wave function is antisymmetrized according to two identical fermions in the system. Each component is expressed by corresponding Jacobi coordinates. For the system the set of the Faddeev equations is presented by two equations for the components U and W [33]: The $NN\bar{K}$ ($s_{NN} = 0$) system is represented by four configurations: ppK^- , npK^- , $np\bar{K}^0$, $nn\bar{K}^0$. The set of pair potentials and masses of particles in Eq. (2) definite each configuration. For the system ppK^- , we have $V_{NN} = v_{pp}$ and $m_N = m_p$, $m_{\bar{K}} = m_{K^-}$. Where v_{pp} is spin-singlet proton-proton potential, m_p is the proton mass, and m_{K^-} is the mass of K^- . The input for the npK^- , $np\bar{K}^0$, $nn\bar{K}^0$ configurations is chosen by similar way using the pp , pn and nn components of spin-singlet nucleon-nucleon potential. The difference in masses of nucleons and masses of kaons are also taken into account.

To describe the spin-singlet NN interaction ($I_{NN} = 1$), we are applying the semi-realistic s -wave Malfliet-Tjon MT I-III potential with the modification from Ref. [34]:

$$V_{NN}(r) = (-513.968\exp(-1.55r) + 1438.72\exp(-3.11r))/r, \quad (3)$$

where the strength (range) parameters are given in MeV (in fm^{-1}). The pp , np and nn potentials are obtained by scaling the potential (3) by the factor γ chosen in [32] as 0.9745, 1, 0.982, respectively.

Particle configurations can be also described in the nuclear NNN system as the nnp , ppn set depending on pair potentials and mass used. The AAB model with two identical particles for the NNN system was proposed in Ref. [32]. The charge symmetry breaking for the nucleon-nucleon potential and the mass difference of proton and nucleon lead to the difference of the nnp and ppn systems (if the Coulomb force is ignored). In particular, one can associate the NNN system with the ^3H nucleus, when spin-singlet nn potential for identical particles is used, and with the ^3H nucleus, when spin-singlet pp potential is used. The detailed description of this model may be found in Ref. [32].

We employ the s -wave Akaishi-Yamazaki (AY) [14] effective potential for $N\bar{K}$ interaction. The potential includes the $N\bar{K}/\pi\Sigma$ coupled-channel dynamics into a

single channel $N\bar{K}$ interaction. The potential are written in the form of one range Gaussian: $V_{N\bar{K}}^{s(t)}(r) = V_0^{s(t)} \exp(-r/b)^2$, where the parameters are depended on pair isospin which defines the singlet (s) and triplet (t) states: $V_0^s = -595.0$ MeV, $V_0^t = -175.0$ MeV, $b = 0.66$ fm. According to Eq. (2), the $N\bar{K}$ potential is the same for all configurations.

We would like to note that a "particle model" which ignores isospins in the charged " ppK^- " kaonic system and defines pair $p\bar{K}^-$, $n\bar{K}^0$, $p\bar{K}^0$ potentials was considered in Ref. [35]. This model motivated by Ref. [28] presents the coupled ppK^- and $np\bar{K}^0$ particle configurations as " $ppK^-/np\bar{K}^0$ cluster". The numerical analysis for such two-level system had shown essential disagreement with recently reported experimental data.

The Coulomb interaction is ignored in our consideration due to its contribution, which can hide the effects related to nucleon-nucleon potentials and particle mass differences.

4. Modeling

4.1 Method

The numerical solution of the Faddeev equations (1) was obtained by the cluster reduction method [36, 37] in which we used the expansion of the components U and W along with the basis of the eigenfunctions of two-body Hamiltonians of the subsystems:

$$U(x, y) = \sum_{i \geq 1} \phi_i^U(x) F_i^U(y), \quad W(x, y) = \sum_{i \geq 1} \phi_i^W(x) F_i^W(y).$$

Here, the functions F_i^U and F_i^W , $i = 1, 2, \dots$ are describe the relative motion of "clusters" in each rearrangement channel $(NN)\bar{K}$ and $N(N\bar{K})$, respectively. The functions F_i^U (F_i^W) depend on the relative coordinate y . The solutions of the two-body Schrödinger equations form complete set of eigenfunctions in the box, $x \subset [0, R_x]$:

$$\left(-\frac{\hbar^2}{2\mu^U} \partial_x^2 + V_{NN}^{sNN=0}(x)\right) \phi_i^U(x) = \epsilon_i^U \phi_i^U(x), \quad \left(-\frac{\hbar^2}{2\mu^W} \partial_x^2 + V_{N\bar{K}}(x)\right) \phi_i^W(x) = \epsilon_i^W \phi_i^W(x),$$

where, μ^U (and μ^W) is reduced mass of the pairs, $\phi_i^U(0) = \phi_i^U(R_x) = 0$ ($\phi_i^W(0) = \phi_i^W(R_x) = 0$), $i = 1, 2, \dots, p$. The parameter R_x is chosen to be enough large to reproduce the binding energy of $N\bar{K}$ subsystem in the isospin singlet channel (see also Ref. [38]). In the presented calculations, $R_x=35$ fm was chosen that provides the convergence of the results of the three-body calculations for $p \leq 70$.

4.2 Numerical results

The results of the calculations for the $NN\bar{K}$ ground state energy are presented in Table 1 for the different particle sets of the $NN\bar{K}$ model. In the table, we show input

Table 1: The energies and masses (in MeV) of the quasi-bound states for different particle configurations of the $NN\bar{K}(S_{NN} = 0)$ system.

Configuration	NN	m_N	$m_{\bar{K}}$	E_2	E_3	$E_2 - E_3$	M
$pp\bar{K}^-$	pp	938.272	493.677	-30.259	-45.7	15.4	2324.5
$np\bar{K}^0$	np	938.920	497.611	-30.867	-46.7	15.8	2328.7
$np\bar{K}^-$	np	938.920	493.677	-30.285	-46.1	15.8	2325.4
$nn\bar{K}^0$	nn	939.565	497.611	-30.893	-46.5	15.6	2330.2
$NN\bar{K}$	np	938.920	495.644	-30.576	-46.5	15.9	2327.0

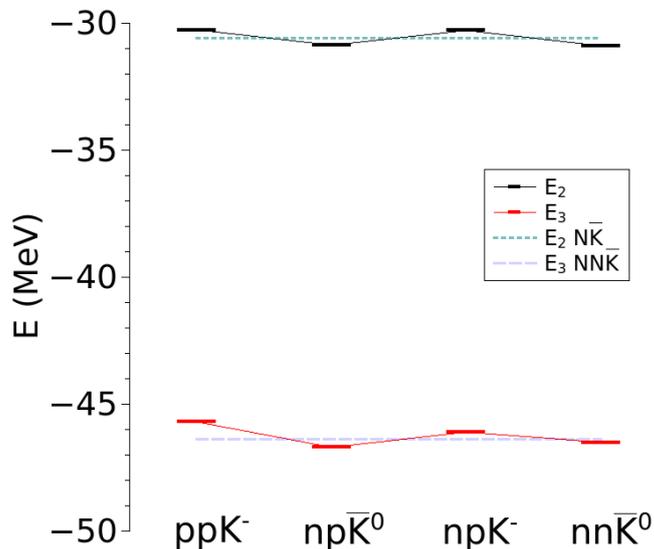


Figure 1: The two and tree-body energies of the particle configurations of $NN\bar{K}$ system. The energy is measured from three-body threshold. The horizontal dashed lines represent the averaged isopin model with results shown in lower row of Tab. 1.

parameters of each particle configuration, nucleon-nucleon potentials and masses of the particles. For the $np\bar{K}^-$ and $np\bar{K}^0$ configurations, we apply the averaged mass on nucleon to keep the AAB model. For more rigorous consideration, the ACB model is needed to use. Here, E_2 and E_3 are two and three-body binding energies of the $N\bar{K}(I = 0)$ and $NN\bar{K}$ systems. The system is bound due to attractive isospin-singlet component ($I = 0$) of the $N\bar{K}$ potential and weak attractive spin-singlet NN potential. The last one does not form a two-body bound state. The E_2 is related to the experimental value for the defect mass about 27 MeV for Λ -hyperon resonance $\Lambda(1405)$ considering as the bound $N\bar{K}$ state [17]. The $E_2 - E_3$ is the binding energy of one of the nucleons relative to a two-body threshold. The M is the mass of a configuration calculated as the sum of free mass particles minus

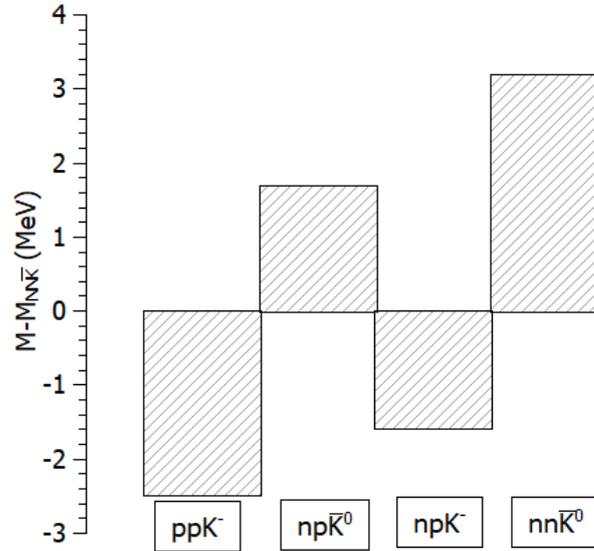


Figure 2: The mass difference $M - M_{NN\bar{K}}$ for particle configurations in the $NN\bar{K}$ ($s_{NN} = 0$) system.

the E_3 (the mass defect) measuring in MeV. Obtained results are comparable with the results of calculations performed within different approaches. For example, calculated value $|E_3|$ reported in Ref. [25] is about 46.6 MeV for phenomenological $\bar{K}N$ potential with two-pole $\Lambda(1405)$ structure (see also Ref. [39]).

The graphical illustrations for the variation of the binding energy and masses for different particle configurations are presented in Figs. 1, 2, and 3. One can see in Fig. 1 that the variation in the three-body energy E_3 is following to the variation of the two-body energy E_2 . Thus, the values $E_2 - E_3$ looks as similar for all particle configurations. The E_3 variations are small comparing to the scale defined by the $E_2 - E_3$ energy. The variations of total masses of the particle configurations are shown in Fig. 2 relatively the total mass of the isospin model $NN\bar{K}$ with averaged nucleon and kaon masses (see Tabl. 1). The mass differences $M - M_{NN\bar{K}}$ are less than 3.5 MeV depending on particle configuration. In other words, the deviation of the masses of particle configurations around the $NN\bar{K}$ ($s_{NN} = 0$) mass is less than 0.15%. Similar variations for three-body energy are shown in Fig. 3 a). The maximal energy difference $E - E_{NN\bar{K}}$ is related to the ppK^- system and evaluated about 0.8 MeV. Thus, this derivation is the uncertainty for $NN\bar{K}$ calculations when the entry channel is not definite. The value represented by relative energy error is not so large and can be evaluated as $0.8/47$, which is about 1.7%.

In Fig. 3 b), we illustrate our calculations using the comparison with calculated results for particle configurations in the NNN system. Here, the results from Ref. [32] are implied, in which a phenomenological potential is used with the corrections to take into account the charge symmetry breaking in the NN interaction. The mass difference of proton and neutron has also been assumed. In Fig. 3, it is seen that the effects related to the charge symmetry breaking and particle mass

differences are stronger in the $NN\bar{K}$ system.

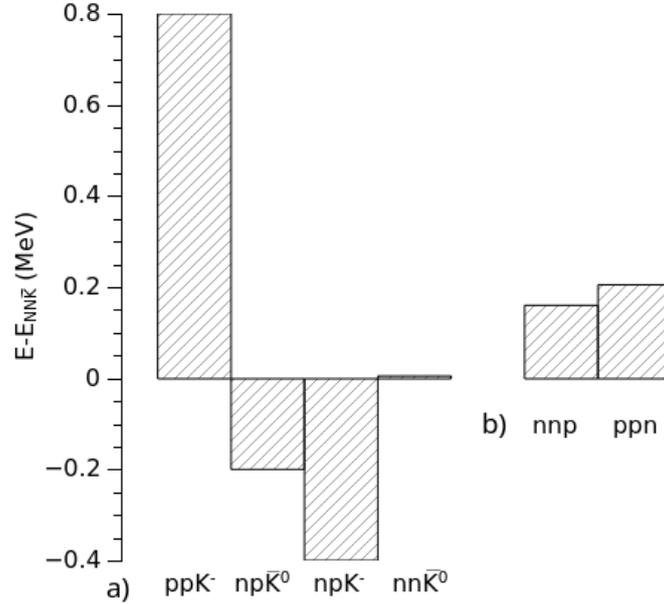


Figure 3: The energy difference a) $E - E_{NN\bar{K}}$ for the particle configurations in $NN\bar{K}$ system, b) $E - E_{NNN}$ for the particle configurations in NNN system.

5. Conclusions

The $NN\bar{K}(s_{NN} = 0)$ bound state problem was formulated by the configuration space Faddeev equation using isospin dependent phenomenological $N\bar{K}$ potential. The model has taken into account nucleons/mesons mass differences and charge dependence of nucleon-nucleon potential. The particle configurations are defined as ppK^- , $np\bar{K}^0$, $nn\bar{K}^0$, npK^- . The three-body calculations are performed to evaluate the mass and energy for each configuration. In particular, the deviation of the masses around the $NN\bar{K}(s_{NN} = 0)$ mass is less than 0.15% of the $NN\bar{K}$ mass and less than 1.7% of the $NN\bar{K}$ energy. Thus, the configuration splitting can be defined as a fine effect, and the isospin $NN\bar{K}$ model is an acceptable approximation for the systems. The Coulomb interaction taken into account may correct the numerical values of the ppK^- and npK^- energy splittings on 0.5 MeV[26]. The configuration splitting effect is comparable to the charge symmetry breaking effect for ${}^3\text{H}$ and ${}^3\text{He}$ nuclei (about 0.6% for energy).

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