



Pauli basis formalism in quantum computations

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Abstract. This article deals with quantum computations in the Pauli basis whose elements are usually identified with the Pauli strings. This approach allows us to represent quantum states, observables, and unitary operators in the unified form of linear combination of Pauli strings, so that all operations can be reduced to the string compositions. Nevertheless a formal justification of the Pauli basis for quantum computations should be based on the strong results of complex linear algebra and the theory of Hilbert spaces. We briefly review the main features of Pauli strings for quantum states and unitary operators, and also the key operations with them, including an algorithm for string compositions and a transformation algorithm from the standard basis to the Pauli basis.

Keywords: quantum computations, Pauli basis

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