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Synthesis of the law of motion control for a swarm of autonomous drones using the method of mobile cellular automata

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Abstract. We consider the construction of motion control system for a swarm of drones using the methods of the elasticity theory from continuum mechanics. For a swarm of drones in the form of a linear chain, the problem of maintaining the initially specified shape throughout the entire time of movement is solved. The theory of mobile cellular automata is applied to construct a mathematical model of the chain of drones. By passage to the limit, equations of longitudinal and transverse oscillations of the chain are obtained, similar to those of the longitudinal vibrations of a rod and the transverse oscillations of a stretched string. The longitudinal and transverse oscillations of the resulting system, resulting from the influence of external perturbations are investigated, as well as the effect of these oscillations on the stability of the swarm configuration.

Keywords: scanning lidars, ToF-scanners, computer vision, mobile cellular automata, swarms of autonomous vehicles, wave equation, motion control
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1. Introduction

Recently, a large number of publications on autonomous motion control of flocks (swarms) of apparatuses (flock agents) of various purposes have appeared, including underwater, sea-based, ground-based, air-based, and space-based ones [3]–[5]. The goal of the proposed research is to develop a methodology and laws for autonomous motion control in a swarm of various-purpose vehicles based on the formalism of moving cellular automata [1], a numerical method of continuum mechanics successfully used for describing the behaviour of granular media, their local deformations and mixing of substance in them [2].

The object of control in this context is a flock (swarm) of relatively simple robots. The instrumental composition of each agent of the swarm includes inertial navigation devices (GPS, GLONASS) and angular velocity sensors. To create an internal navigation field of the swarm, wide-angle range finders, scanning lidars, and ToF-scanners are used. The equipment of each agent of the swarm includes an on-board computer, which generates control signals applied to the actuators according to estimates from the outputs of the customised on-board model. The on-board custom model itself is an adaptive observer with the input noisy "raw" signals, generated in the measuring channels of the sensor equipment. The choice of actuators that create a field of control actions depends on the swarm deployment. The term flock (swarm) is applied to a grouping of robots consisting of at least 20 agents and, as a result, the goal must be achieved within the paradigm of group robotics. Group robotics is a new approach to coordinating the systems of many robots, which consist of a large number of mostly simple physical robots. It is assumed that the desired collective behaviour arises from the interaction of robots with each other and their interaction with the environment. The methodology and laws of the swarm control in this work are based on the mathematical methods developed in the theory of mobile cellular automata. The method of movable cellular automata (MCA) is a relatively new method of computational mechanics of a deformable solid, based on a discrete approach. It combines the advantages of the method of classical cellular automata and the method of discrete elements. Here (possibly for the first time), the MCA formalism developed by S.G. Psakhie, G.P. Ostermeier, A.I. Dmitriev, E.V. Shilko, A.Yu. Smolin, and S.Yu. Korostelev [1] is applied to create an adaptive decentralized control system for a swarm of robots with customized on-board models of targeted group behaviour. The main idea of the proposed approach is to apply numerical methods of continuum mechanics to the creation of motion and configuration control algorithms for packs (swarms) of relatively simple intelligent robots. This idea is naturally born from a comparison of the structures of research objects in continuum mechanics and control objects in the theory of motion control of systems with distributed parameters. As will be shown below, in the simplest cases, a swarm can be described (by passing to the limit, as the cell size tends to zero) using the same partial differential equations as the elastic-plastic body.

2. Setting of the problem

The concept of the movable cellular automata method is based on the introduction of a new type of state: any pair of automata can have two types of states, bound (when the automata interact) and unbound (when they are considered non-interacting).

Following [1], consider a pair of automata ij. Let $r^{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ be the distance between the centres of two elements, then $h^{ij} = r^{ij} - r_0^{ij}$ is the overlap parameter of a pair of automata (Figure 1). Here r_0^{ij} is the distance between the centres of the elements in the initially given configuration of the swarm (the sum of the radii of the zones of influence of the elements). The bound state of a pair of automata will be determined by the relation $h^{ij} < h^{ij}_{\max}$, where $h^{ij}_{\max}$ is a certain fixed value, and the unbound state is determined by the relation $h^{ij} > h^{ij}_{\max}$.

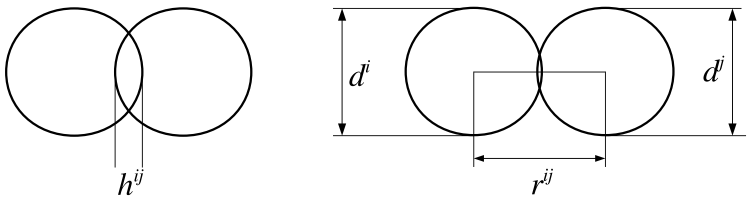

Figure 1: Overlapping of a pair of automata.

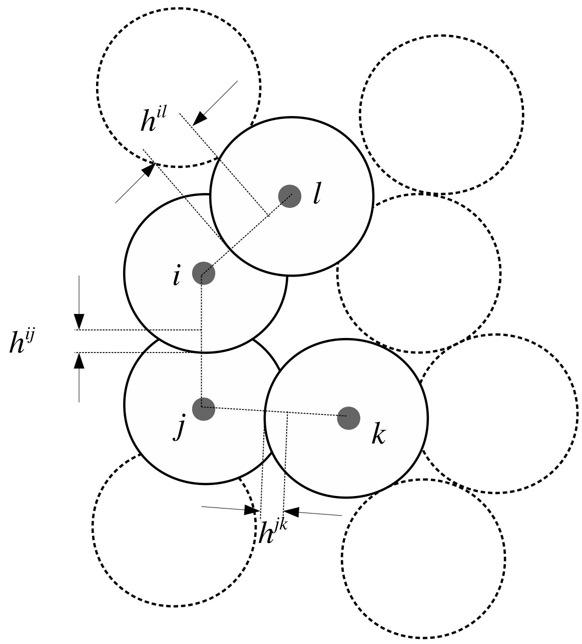

Figure 2: An ensemble of interacting automata.

If there are three or more automata in the system under consideration, the state of the pair ij will be also affected by the state of other interacting pairs (Figure 2).

When constructing the control system, we will use analogies from the theory of elasticity, i.e., to control swarms we will use analogues with the elastic interaction between particles of a continuous medium.

At the initial stage, we consider the case when cellular automata are located along a straight line. In this case, the bound states of automata are formed by the conditions of the neighbourhood along a linear chain of automata, i.e., the i-th automaton interacts with the $(i-1)$-th and $(i+1)$-th automata.

The goal of controlling a linear chain is to maintain the order, i.e., all elements of the swarm should be located along a straight line at specified distances from each other. The chain as a whole can perform translational and rotational motion, depending on external perturbations.

Let N be the number of all elements in the swarm, a be the distance between the centres of neighbouring elements in the unperturbed state. The initial unperturbed position of the chain is defined along the x axis, and the coordinate of the i-th element is given by the formula $x_{i0} = ia$, $i = \overline{1, N}$. We consider the motion in the xy plane, i.e., the coordinate z for all elements of the swarm in the process of movement is zero.

Denote the current position and speed of the i-th element of the swarm as

$$\mathbf{r}_i = \begin{vmatrix} x_i & y_i & 0 \end{vmatrix}^T, \quad \mathbf{v}_i = \begin{vmatrix} v_{ix} & v_{iy} & 0 \end{vmatrix}^T.$$

The system of equations describing the movement of the swarm has the form

$$m_i \frac{d\mathbf{v}_i}{dt} = \mathbf{F}_i, \quad i = \overline{1, N}, \tag{1}$$

where m_i is the mass of the i-th element of the swarm, $\mathbf{F}_i$ is the total force acting on the i-th element. The force $\mathbf{F}_i$ consists of the control part $\mathbf{U}_i$, which supports the order of the swarm, and the external perturbing force $\mathbf{F}_i^{\text{ext}}$. For modelling, it is convenient to reduce system (1) to a system of first-order equations by introducing the state vector

$$\mathbf{s} = \begin{vmatrix} \mathbf{r}_1^T & \dots & \mathbf{r}_N^T & \mathbf{v}_1^T & \dots & \mathbf{v}_N^T \end{vmatrix}^T.$$

Then the system of equations (1) takes the form:

$$\frac{d\mathbf{s}}{dt} = \begin{vmatrix} \mathbf{v}_1^T & \dots & \mathbf{v}_N^T & \mathbf{F}_1^T/m_1 & \dots & \mathbf{F}_N^T/m_N \end{vmatrix}^T.$$

To keep order in the swarm, it is necessary to specify the control forces $\mathbf{U}_i$, which include forces acting in the longitudinal direction relative to the chain, and forces acting in the transverse direction.

3. Formation of the navigation field and the field of control forces inside the swarm

The essence of the proposed approach to the design of the swarm control system is as follows. The structure of the movement control system of the swarm must have a two-tier architecture.

The lower level of control is completely decentralised. It is entrusted with the solution of the task “to keep the line”, that is, to keep the geometrical configuration of the swarm close to the required (reference) one. This configuration, in turn, is defined by the upper level of the swarm control system. The problem is solved by MCA mathematical methods. Each agent included in the grouping is surrounded by a virtual cell, which, within the framework of the developed MCA formalism, is treated as an element of an ensemble of mobile cellular automata. The concept of the MCA method is based on the introduction of a new type of state within the framework of the paradigm of classical cellular automata — the state of a pair of automata. This made it possible to take a fundamentally important step, namely, to introduce into consideration the overlap of a pair of automata as an adjustable parameter. As a result, in the framework of the proposed concept, there is no need to follow each automaton for the dynamic behaviour of all other agents of the swarm, except for devices located in the immediate vicinity of its virtual cell. This can significantly reduce the load on the computer network of the pack. The virtual cell itself is formed by means of laser location, radar, echolocation and sonar (depending on the environment of the swarm), creating a local navigation field, its dimensions and geometric shapes being set by the upper level of the control system. The overlap parameter of two neighbouring cells can be interpreted in terms of the MCA as a local “compression deformation” of the swarm structure, and in case of a predetermined distance between neighbouring automata as a local “tensile deformation”. The mechanical evolution of the “longitudinal deformations” for each cellular automaton is determined by the solutions of the Newton equations programmed in the on-board tuneable models of the dynamic behaviour of each agent of the swarm, and is further adjusted by the readings of the relative position and speed sensors. The a posteriori estimates of the relative position and velocity obtained in this way are the initial information for the formation of a control action for the countering of the “longitudinal deformations” that arise. Similarly, in the framework of the MCA method, “angular deformations” of local “torsion” and “bending” are introduced, but the evolution of these deformations is determined by solving the Euler equations (not considered in this article) with subsequent correction of information from angular velocity sensors and azimuth angles. The on-board dynamic behaviour model, implemented in the on-board microcomputer of each of the agents of the pack, functions in real time. It is a collection of reference and custom models of the desired behaviour of the pack. Depending on the required behaviour of the pack and the mission assigned to it, the borders of the cell, in the geometric centre of which each agent of the pack is located, may have a different geometric shape: regular and deformed Platonic solids, ellipsoids and spheres. In the proposed approach, the shape and size of each virtual cell can be changed quite simply. Rescaling can be done autonomously, using information generated in the loop of the top-level control system, about the threat, obstacles to the movement of the group, or the approach of an unfriendly object.

The control forces $\mathbf{U}_i$ are generated according to the law of feedback, as a function of the relative position and relative speed of only neighbouring agents of the swarm.

Next, we consider simplified examples of the implementation of the proposed methodology for the synthesis of the law of swarm motion control.

4. Longitudinal oscillations

Consider the oscillations of the elements of the swarm, assuming that movements occur only along the x axis (Figure 3), i.e., we set y_i and v_{iy} equal to zero. The position of the i-th element is written in the form $x_i = x_{i0} + u_{ix}$, where u_{ix} is the displacement of the element from the initial unperturbed position.

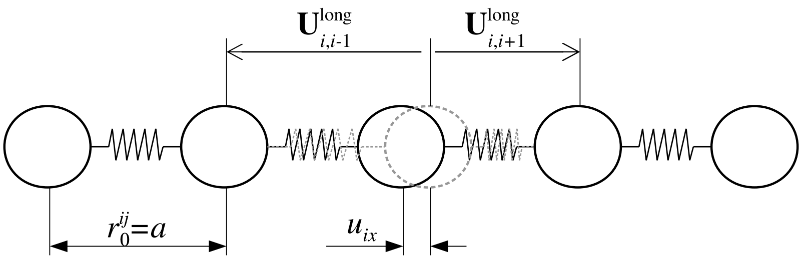

Figure 3: To the derivation of the equation describing longitudinal oscillations of the chain.

In order to maintain constant distances between neighbouring elements, we define the control force acting on the i-th element from the neighbours by the formula

$$U_{ij}^{\text{long}} = \frac{k}{a}\left(u_{jx} - u_{ix}\right), \quad j = i+1, i-1, \tag{2}$$

where k is the coefficient of "stiffness" of the control in the longitudinal direction. Therefore, the equation of motion of the i-th element is written as

$$m_i \frac{d^2 u_{ix}}{dt^2} = U_{i,i-1} + U_{i,i+1} = \frac{k}{a}\left(u_{i-1,x} + u_{i+1,x} - 2u_{ix}\right).$$

Passing to the limit as $a \to 0$ (or when the length of the chain tends to infinity) and considering that $m_i = \lambda a$, where λ is the mass per unit length of the linear chain, we get the wave equation

$$\lambda \frac{\partial^2 u_x}{\partial t^2} = k \frac{\partial^2 u_x}{\partial x^2}, \qquad x \in (0, l), \tag{3}$$

where l is the length of the chain.

Equation (3) is similar to the equation of rod oscillations in the longitudinal direction [6]. Indeed, if a rod of length l with a Young's modulus E, a cross section of the rod S and a material density ρ is divided into small parts of length a, then $m_i = \rho S a$ and $k = ES$ according to Hooke's law. After passing to the limit, we get

$$\rho S \frac{\partial^2 u_x}{\partial t^2} = ES \frac{\partial^2 u_x}{\partial x^2}, \qquad x \in (0, l).$$

It is known that the propagation velocity of a longitudinal wave in a rod is $v_s = \sqrt{E/\rho}$, in the case of a linear chain this will be

$$v_s = \sqrt{k/\lambda}. \tag{4}$$

Now let us perform numerical simulation with the parameters $a = 1$ m, $m_i = 1$ kg, $k = 1$ N, $N = 10$. The force that maintains a constant distance between the elements is set in the form

$$\mathbf{U}_i^{\text{long}} = \mathbf{U}_{i,i-1}^{\text{long}} + \mathbf{U}_{i,i+1}^{\text{long}},$$

where

$$\mathbf{U}_{ij}^{\text{long}} = ch^{ij} \frac{\mathbf{r}_j - \mathbf{r}_i}{r^{ij}}, \qquad j = i-1, i+1,$$

the coefficient $c = k/a$ and the overlap parameter $h^{ij} = r^{ij} - a$ (Figure 3).

Assume that at the initial moment of time the elements are in the undeformed state with zero velocities. Let us impart to the left element a momentum produced by a force of 1 N that acts during 1 s. Figure 4 shows the positions of elements of the swarm at times $t = 1$ s and $t = 6$ s.

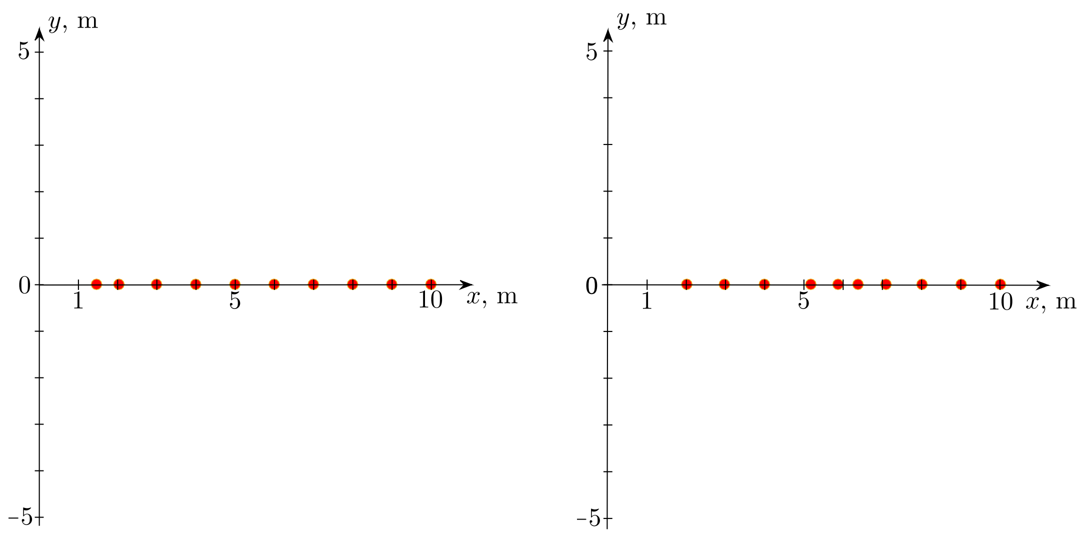

Figure 4: Propagation of a single pulse at $t = 1$ s (left) and $t = 6$ s (right).

As seen from Figure 4, a wave really appears, traveling at a speed of approximately 1 m/s, which is consistent with Equation (4) derived above. After the wave reaches the right-hand end of the chain, it is reflected from the free edge and runs to the left. Because of such reflections, the chain will crawl to the right, which is consistent with the law of momentum change. Similar processes can be observed in heavy car traffic.

Let us consider one more illustration of the obtained passage to the limit. Now we fix the 1-st and the N-th elements of the chain, i.e., put $\mathbf{F}_i = 0$, $i = 1, N$. From the mathematical physics, it is known that Equation (3) with fixed edges has eigenfunctions [7]

$$u_n(x) = \sin \frac{\pi n x}{l}.$$

Let us set the initial position of the elements of the swarm by the formula

$$\mathbf{r}_{i0} = \left| x_{i0} + \sin\left(\pi(i-1)/(N-1)\right) \quad 0 \quad 0 \right|^T .$$

The initial velocities of all elements are assumed zero. The simulation shows that under the influence of the forces of the longitudinal interaction between the elements, the system will begin to oscillate along the x-axis, keeping the shape of a sinusoid. Figure 5 shows the arrangement of elements at times $t = 0$ s and $t = 9$ s.

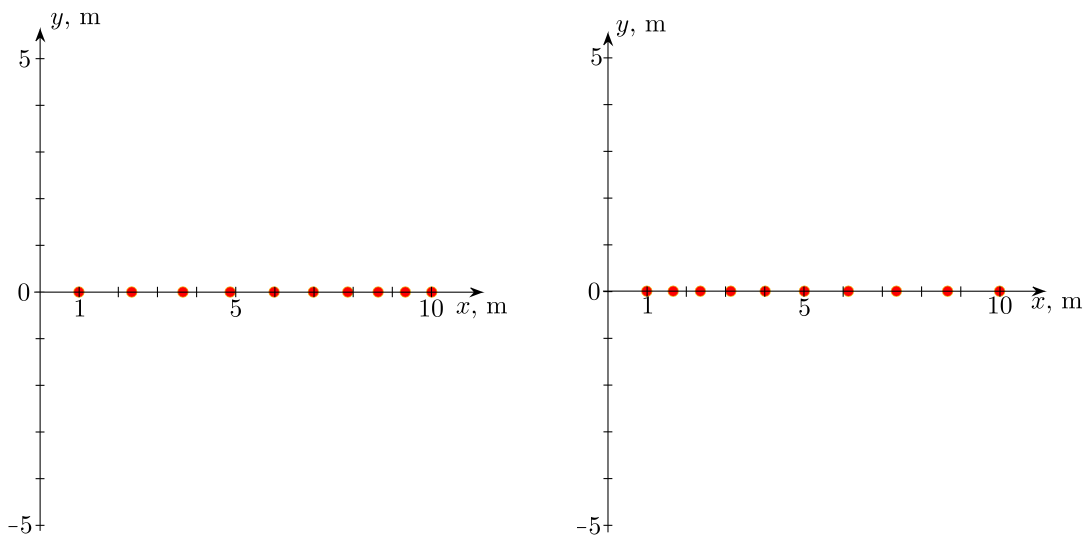

Figure 5: Oscillations of the system with the specified initial position according to its own shape at $t = 0$ s (left) and $t = 9$ s (right).

It can be seen from Figure 5 that the oscillation period τ is approximately equal to 18 s. Accordingly, the wave propagation velocity $v_s = 2l/\tau = 18/18 = 1$ m/s, which agrees with Equation (4).

Thus, the equations of continuum mechanics allow not only to implement the control of a swarm of drones, but also to qualitatively evaluate the behaviour of the swarm as it moves.

The obtained results show that non-damping longitudinal oscillations appear in the system. Since this consumes control resources, it is necessary to introduce forces that allow the resulting oscillations to be damped to zero in a finite period of time. Thus, it is necessary to complete the control forces $\mathbf{U}_{ij}^{\text{long}}$ with a term proportional to the time derivative of the relative distance between the i-th and j-th elements, i.e., with damping forces. This damping force is expressed as

$$\mu c \frac{dh^{ij}}{dt}, \qquad j = i+1, i-1,$$

where μc is the longitudinal damping coefficient.

As a result, the longitudinal control force will take the form

$$\mathbf{U}_{ij}^{\mathrm{long}} = c\left(h^{ij} + \mu \frac{dh^{ij}}{dt}\right)\frac{\mathbf{r}_j - \mathbf{r}_i}{r^{ij}}, \qquad j = i+1, i-1.$$

In the simulation, we use the expression

$$\frac{dh^{ij}}{dt} = \frac{(\mathbf{r}_j - \mathbf{r}_i, \mathbf{v}_j - \mathbf{v}_i)}{r^{ij}}, \qquad j = i+1, i-1.$$

Figure 6 shows the arrangement of elements at times $t = 1$ s and $t = 800$ s.

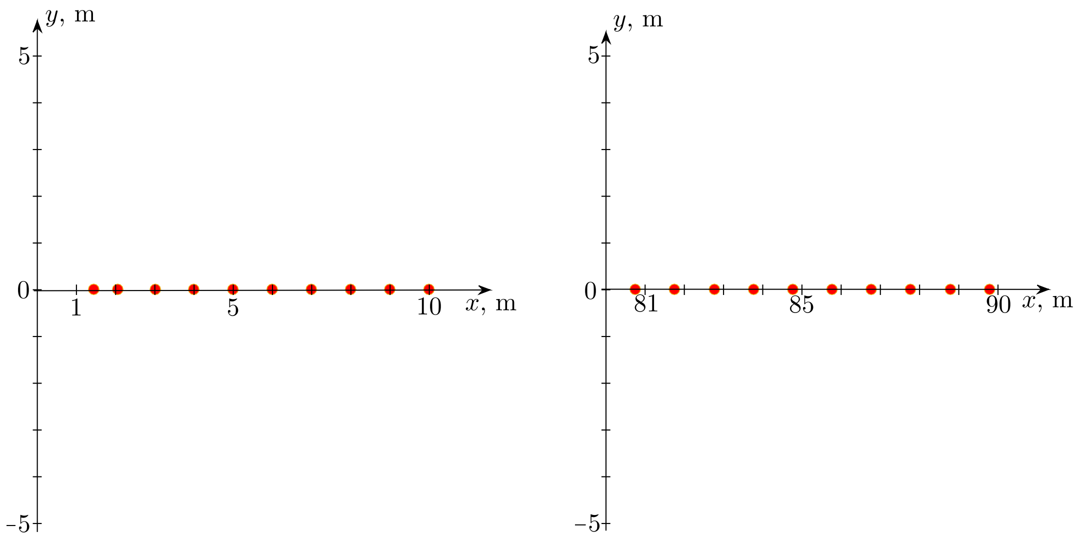

Figure 6: Propagation of a single pulse at $t = 1$ s (left) and $t = 800$ s (right) in the damped system ($\mu = 0.1$ s).

It turns out that in 800 seconds the system completely dampens the longitudinal vibrations. Since the main task is to describe the movement of the chain in the plane, it is not enough to maintain constant distances between the elements of the chain; it is necessary to introduce the control in the transverse direction, which leads to the appearance of transverse oscillations in addition to the longitudinal ones.

5. Transverse oscillations

As in the previous case, we consider the oscillations of the elements of the swarm, but the displacements occur along the y-axis ($v_{ix} = 0$). Let u_{iy} be the displacement of the i-th element relative to the unperturbed position along the y-axis, $y_i = y_{i0} + u_{iy}$ (Figure 7).

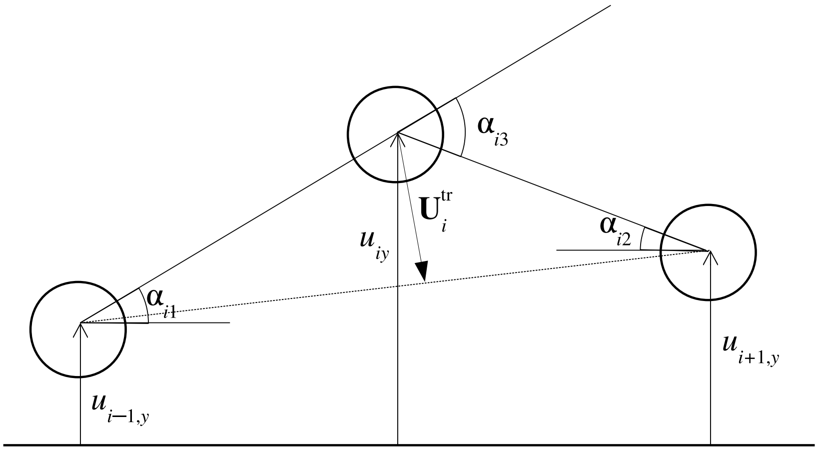

Figure 7: To the derivation of the equation for transverse oscillations of the chain.

In order to stabilize the chain in the transverse direction, let us set the control force $U_i^{\rm tr}$ acting on the i-th element in the direction along the y-axis, in the form

$$U_i^{\rm tr} = T\alpha_{i3},$$

where T is the coefficient of the control "stiffness"; α_{i3} is the angle formed by the directions $i-1, i$ and $i, i+1$.

Since $\alpha_{i3} = \alpha_{i2} - \alpha_{i1}$, where α_{i1}, α_{i2} are the angles formed by the directions $i-1, i$ and $i, i+1$ with the x-axis and

$$\alpha_{i1} \approx \frac{u_{iy} - u_{i-1,y}}{a}, \qquad \alpha_{i2} \approx \frac{u_{i+1,y} - u_{iy}}{a},$$

the equation of motion of the i-th element in the transverse direction has the form

$$m_i \frac{d^2 u_{iy}}{dt^2} = T\left(\frac{u_{i+1,y} - u_{iy}}{a} - \frac{u_{iy} - u_{i-1,y}}{a}\right) = T\frac{u_{i+1,y} - 2u_{iy} + u_{i-1,y}}{a},$$

which after passing to the limit $a \to 0$, with $m_i = \lambda a$ taken into account, yields the equation

$$\lambda \frac{\partial^2 u_y}{\partial t^2} = T\frac{\partial^2 u_y}{\partial x^2}, \qquad x \in (0, l). \tag{5}$$

The obtained equation resembles the oscillation equation of a stretched string with tension T [7]. Thus, the control we have constructed introduces an artificial "tension", which allows stabilising the position of the swarm elements in the transverse direction. Let simulate the movement of a linear chain coupled by the introduced transverse "returning" forces. For simulation we take the number of elements $N = 100$, $a = 1$ m, $m_i = 1$ kg, $T = 1$ N and set the initial position of the elements according to Figure 8 (left). The initial velocities of all elements are assumed zero. The returning force acting on the i-th element in the vector form is written as

$$\mathbf{U}_i^{\mathrm{tr}} = T\alpha_{i3}\mathbf{e}_{ui}, \tag{6}$$

where $\mathbf{e}_{ui}$ is a unit vector directed from the i-th element to the middle of the segment connecting the $(i-1)$-th and the $(i+1)$-th elements.

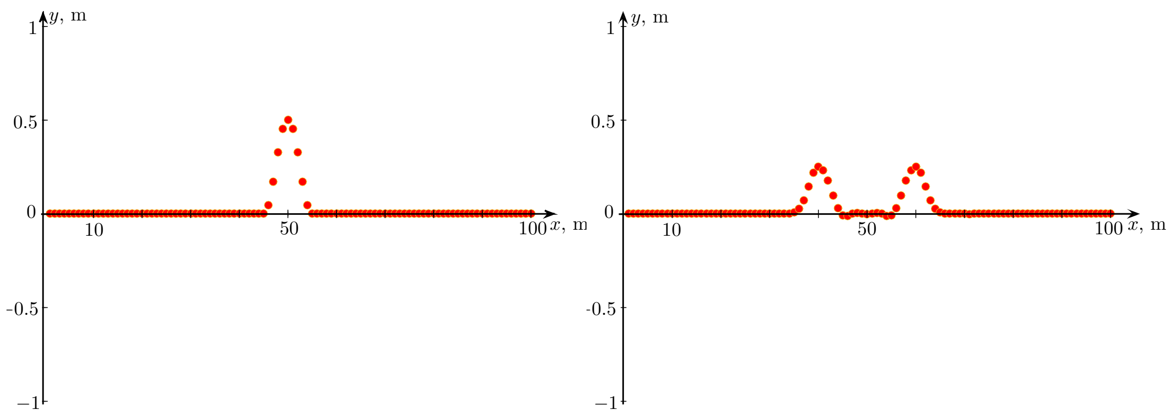

Figure 8: Position of the elements of the system at $t = 0$ s (left) and $t = 10$ s (right).

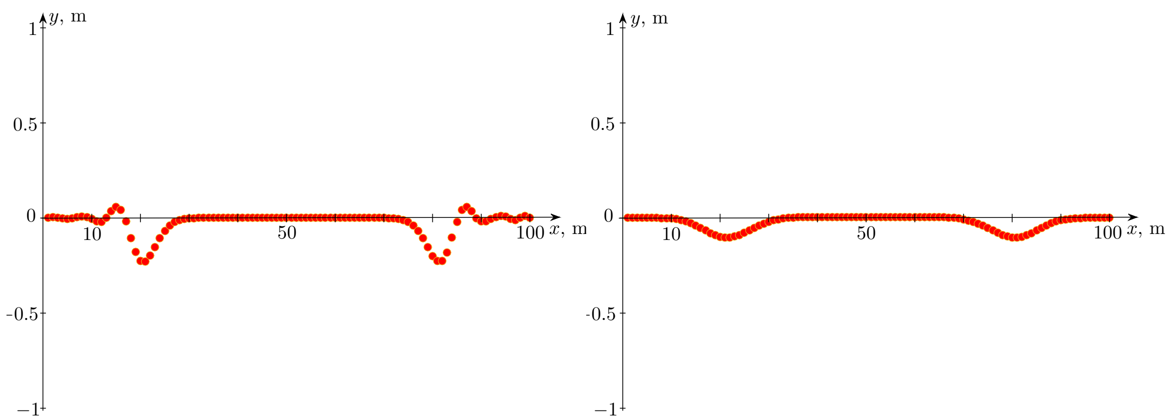

Figure 9: Effect of damping term with $\varepsilon = 0$ s (left) and $\varepsilon = 0.2$ s (right) on the position of the elements of the chain at $t = 70$ s.

As seen from the simulation results (Figure 8, right), two differently directed waves arise, moving with the same speed, approximately 1 m/s, which is consistent with the well-known formula $v_s = \sqrt{T/\lambda}$ for the wave propagation velocity in the equation of motion of the string.

In the case of transverse oscillations, the damping is proportional to the rate of α_{i3} variation:

$$\varepsilon T \frac{d\alpha_{i3}}{dt},$$

where εT is the transverse damping coefficient. Adding this term to Equation (6), we obtain the final expression for the transverse control force

$$\mathbf{U}_i^{\mathrm{tr}} = T\left(\alpha_{i3} + \varepsilon\frac{d\alpha_{i3}}{dt}\right)\mathbf{e}_{ui}.$$

For comparison, we present the simulation results for the previous case in the absence of damping (Figure 9, left) and with the coefficient $\varepsilon = 0.2$ s (Figure 9, right).

6. Modelling a system controlled in longitudinal and transverse direction

Combining the control over the separations between the elements and the chain deviation from a given straight line form, we obtain the equations of motion in the form

$$m_i\frac{d\mathbf{v}_i}{dt} = \mathbf{U}_i^{\mathrm{long}} + \mathbf{U}_i^{\mathrm{tr}} + \mathbf{F}_i^{\mathrm{ext}}, \qquad i = \overline{1, N},$$

where $\mathbf{U}_i^{\mathrm{long}}$ are the longitudinal control forces; $\mathbf{U}_i^{\mathrm{tr}}$ are the transverse control forces; $\mathbf{F}_i^{\mathrm{ext}}$ are the external perturbing forces.

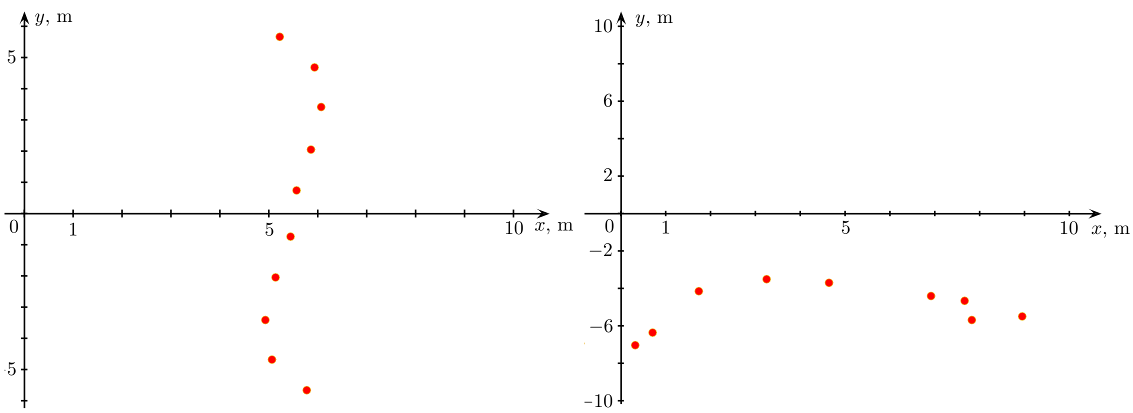

Figure 10: Position of the elements of a chain controlled in the transverse and longitudinal directions without damping at time $t = 12$ s (left) and $t = 380$ s (right).

As an example, consider the case of a chain with the number of elements $N = 10$ and the parameters $a = 1$ m, $m_i = 1$ kg, $T = 1$ N, $k = 1$ N. External forces are applied to the end-point elements of the chain during 1 s and are equal to

$$\mathbf{F}_1^{\mathrm{ext}} = \begin{vmatrix}0 & -1 & 0\end{vmatrix}^T, \quad \mathbf{F}_N^{\mathrm{ext}} = \begin{vmatrix}0 & 1 & 0\end{vmatrix}^T,$$

i.e., we apply to the end-point elements single impacts in opposite transverse directions.

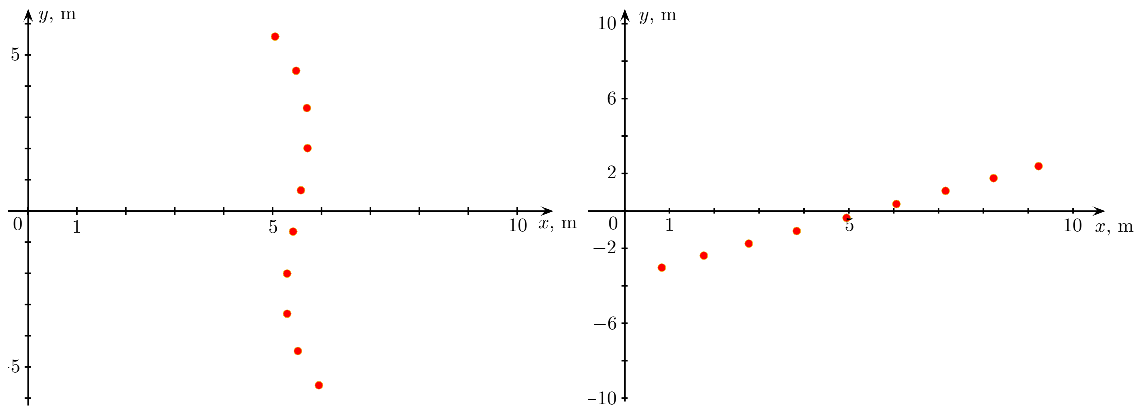

Figure 11: Position of the elements of a chain controlled in the transverse and longitudinal directions in the presence of damping ($\mu = 0.2$ s, $\varepsilon = 0.2$ s) at time $t = 12$ s (left) and $t = 380$ s (right).

First, we present the simulation results without damping, i.e., we set the coefficients μ,ε equal to zero (Figure 10). The results show that the longitudinal and transverse oscillations can take part in a complex interference interaction. At time $t = 12$ s, the chain retains its linear order, and at time $t = 380$ s, the elements lose their initial position relative to each other, which is unacceptable. Now we proceed to a simulation with the damping of the longitudinal and transverse oscillations (Figure 11). The comparison of the results obtained clearly shows that even for a short period of time $t = 12$ s (see Figures 10, 11 (left)), the damping terms contribute significantly to the behaviour of the chain. After 380 s from the start of the movement, the longitudinal and transverse oscillations are practically absent.

7. Conclusions

For swarms in the form of linear chains, a system of elastic control is introduced between the elements of the swarm, which makes it possible to keep linear order. The control is introduced both in the longitudinal direction, which controls the relative distance between adjacent elements of the chain, and in the transverse direction, which does not allow the elements of the swarm to deviate from the specified shape of the chain. By passing to the limit, with the number of swarm elements tending to infinity the wave equations describing the oscillations of the swarm are obtained. Using these equations, a qualitative analysis of the behaviour of the swarm under various external perturbations was carried out. The simulation shows the possibility of implementing control algorithms based on the methods of continuum mechanics. It is shown that for a stable movement of the swarm, the introduction of damping of elastic oscillations is required.

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