



Alternative proof on the crossing number of $K_{2,3,n}$

Michal Staš

Department of Mathematics and Theoretical Informatics, Faculty of Electrical Engineering and Informatics Technical University of Košice, Letná 9, 042 00 Košice, Slovak Republic

e-mail: michal.stas@tuke.sk

Abstract. The main aim of the paper is to give the crossing number of join product $G + D_n$ for the connected graph G of order five isomorphic with the complete bipartite graph $K_{2,3}$, where D_n consists on n isolated vertices. The proof of the crossing number of $K_{2,3,n}$ was published by a partially unclear discussion of cases by Asano in [1]. In our proof, it will be used an idea of cyclic permutations and their combinatorial properties. Finally, by adding one edge to the graph G , we are able to obtain the crossing number of the join product with the discrete graph D_n for one new graph.

Keywords: graph, drawing, crossing number, join product, cyclic permutation

MSC numbers: 05C10, 05C38

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