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Dynamical behavior of a fractional-order eco-epidemiological model with modified Leslie-Gower Holling-type II schemes

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Abstract. In this paper, we introduce a fractional-order eco-epidemiological model with modified Leslie-Gower Holling-type II schemes. We show the solutions of the model are non-negative, and also give a detailed local asymptotical stability analysis of the biologically feasible equilibria. Numerical simulations are presented to illustrate the results.

Keywords: fractional order, eco-epidemiology, local stability

MSC numbers: 92D30, 26A33

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1. Introduction

Dynamical systems combining interacting species with epidemiology known as eco-epidemiology. Many researchers have considered the eco-epidemiological models [1, 2, 3, 4, 5]. In [2], Wang et al considered an eco-epidemiological predator-prey model with stage-structure and latency. In [4, 5], Zhou et al considered eco-epidemiological models with delay.

In this paper, we will study a fractional-order eco-epidemiological model based the model of Zhou et al [6] by adding the assumption that infected prey will get recovery at constant rate. We make the following assumptions for our model (1):

(H_1) We assume that the total prey population is divided into two classes, namely susceptible prey denoted by $S(t)$ and the infected prey denoted by $I(t)$. $y(t)$ is the sizes of predator population.

(H_2) We assume that A is the constant recruitment rate in the prey species. The natural death rates of susceptible prey and infected prey are μ_1 and μ_2, respectively.

(H_3) We assume that the disease is spread among the prey species only and the disease is not genetically inherited, and an infected prey will get recovery at constant rate γ. The incidence is assumed to be the simple mass action incidence βSI, where $\beta > 0$ is the transmission rate of the disease in the prey.

(H_4) Base on the fact that the infected individuals are less active and be caught more easily [7] or the behavior of the infected individuals is modified [7], we assume that predator can distinguish between infected and susceptible prey and the predator eats only the infected prey. And we assume that the functional response of the predator to the prey density is modified Leslie-Gower Holling-type II schemes (see [8, 9, 10]). The predator has a growth rate constant $a_2 > 0$. The maximum value of the per capita rate of I due to y is c_1, and the maximum value of the per capita rate of y due to I is c_2. The extent to which environment protection to prey I (respectively, to the predator y) is k_1 (respectively, k_2).

$$\begin{cases} \dfrac{dS(t)}{dt} = A - \beta S(t)I(t) - \mu_1 S(t) + \gamma I(t), \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - \mu_2 I(t) - \dfrac{c_1 I(t)y(t)}{I(t)+k_1} - \gamma I(t), \\ \dfrac{dy(t)}{dt} = (\alpha_2 - \dfrac{c_2 y(t)}{I(t)+k_2})y(t). \end{cases} \tag{1}$$

Fractional calculus is an area of mathematics that addresses generalization of the mathematical operations of differentiation and integration to arbitrary (non-integer) order. The behavior of most biological systems has memory or after-effects. The fractional order systems are more suitable than integer-order in biological modelling due to the memory effects [11]. In the recent years, fractional calculus has played a very important role in many fields such as mechanics, electricity, biology [12, 13, 14, 15, 16].

Considering the fractional derivatives in the sense of Caputo derivative, and assuming $0 < \alpha \leq 1$, we have the following fractional order eco-epidemiological model corresponding to the model (1):

$$\begin{cases} D_t^{\alpha}S(t) = A - \beta S(t)I(t) - \mu_1 S(t) + \gamma I(t), \\ D_t^{\alpha}I(t) = \beta S(t)I(t) - \mu_2 I(t) - \dfrac{c_1 I(t)y(t)}{I(t)+k_1} - \gamma I(t), \\ D_t^{\alpha}y(t) = (\alpha_2 - \dfrac{c_2 y(t)}{I(t)+k_2})y(t). \end{cases} \tag{2}$$

The meaning of the parameters are similar to system (1). System (2) will be analyzed with the following initial conditions

$$S(0) = S_0 \geq 0,\ I(0) = I_0 \geq 0,\ y(0) = y_0 \geq 0. \tag{3}$$

Denote

$$\mathbb{R}^3_+ = \{((S, I, y)) \in \mathbb{R}^3, S \geq 0, I \geq 0, y \geq 0\}.$$

This paper is organized as follows. In Section 2, some useful definitions and lemmas are presented. A detailed analysis on local stability of equilibrium of the system (2) is carried out in Section 3. Simulations and numerical results are given in Section 4. Conclusions in Section 5 close the paper.

2. Preliminaries

In order to study dynamical behavior of the system (2), we firstly present the definition of fractional-order integration and fractional-order differentiation and some useful lemmas.

There are different forms of definitions of fractional order derivatives, such as, Riemann-Liouville fractional derivative, Caputo fractional derivative, Atangana-Baleanu derivative, Riesz derivative, and so on. It should be pointed out that applied problems require definitions of fractional derivatives allowing the utilization of physically or biology interpretable initial conditions. In fact, Caputo's fractional derivative exactly satisfies these demands.

Hence, in this paper, we will use Caputo's definition, due to its convenience for initial conditions of the differential equations.

Definition 1. [11] *The fractional integral of order $\alpha > 0$ of a function $f : \mathbb{R}^+ \to \mathbb{R}$ is given by*

$$\mathcal{I}^{\alpha} = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt$$

provided the right side is pointwise defined on $\mathbb{R}^+$. Here and elsewhere in this paper, Γ denotes the Gamma function.

Definition 2. [11] *The Caputo fractional derivative of order $\alpha \in (n-1, n)$ of a continuous function f is given by*

$$D_t^{\alpha} f(x) = \mathcal{I}^{n-\alpha} D^n f(x), \quad D = \frac{d}{dt}.$$

In particular, when $0 < \alpha < 1$, we have

$$D_t^{\alpha} f(x) = \frac{1}{\Gamma(1-\alpha)} \int_0^x \frac{f'(t)}{(x-t)^{\alpha}} dt.$$

Lemma 1. (Generalized Mean Value Theorem [17]) *Suppose that* $f(x) \in \mathbb{C}[a,b]$ *and* $D_a^{\alpha} f(x) \in \mathbb{C}(a,b]$, *for* $0 < \alpha \leq 1$, *then we have*

$$f(x) = f(a) + \frac{1}{\Gamma(\alpha)} (D_a^{\alpha} f)(\xi)(x-a)^{\alpha}$$

with $a \leq \xi \leq x$, $\forall x \in (a,b]$.

Lemma 2. *Suppose that* $f(x) \in \mathbb{C}[a,b]$ *and* $D_a^{\alpha} f(x) \in \mathbb{C}(a,b]$, *for* $0 < \alpha \leq 1$. *If* $D_a^{\alpha} f(x) \geq 0$, $\forall x \in (a,b)$, *then* $f(x)$ *is nondecreasing for each* $x \in [a,b]$. *If* $D_a^{\alpha} f(x) \leq 0$, $\forall x \in (a,b)$, *then* $f(x)$ *is nonincreasing for each* $x \in [a,b]$.

Lemma 3. [13] *The equilibrium* (x, y) *of the following frictional-order differential system*

$$\begin{cases} D_t^{\alpha} x(t) = f_1(x,y), D_t^{\alpha} y(t) = f_2(x,y), \alpha \in (0,1], \\ x(0) = x_0, y(0) = y_0 \end{cases}$$

is locally asymptotically stable if all the eigenvalues of the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix}$$

evaluated at the equilibrium (x, y) *satisfy the following condition:*

$$|\arg(\lambda)| > \frac{\alpha\pi}{2}.$$

3. Dynamical behavior of model (2)

In this section, we will discuss the dynamical behavior of system (2).

Theorem 1. *There is a unique solution* $X(t) = (S, I, y)^{\top}$ *to system* (2) *with initial condition* (3) *on* $t \geq 0$ *and the solution will remain in* $\mathbb{R}_+^3$.

Proof. The existence and uniqueness of the solution of (2)-(3) in $(0, +\infty)$ can be obtained from Theorem 3.1 and Remark 3.2 in [18]. In the following, we will show that the domain $\mathbb{R}_+^3$ is positively invariant. Firstly, we prove $S(t) \geq 0$, $\forall t \geq 0$, assuming $S(0) > 0$ for $t = 0$. Suppose that $S(t) \geq 0$, $\forall t \geq 0$ in not true. Then, there exists a $t_1 > 0$ such that $S(t) > 0$ for $t > t_1$. From the first equation of system (2), we have

$$D_t^{\alpha} S(t)|_{t=t_1} = A > 0.$$

According Lemma 1, we have $S(t_1^+) > 0$, which contradicts $S(t_1^+) < 0$, i.e., $S(t) < 0$ for $t > t_1$. Therefore, we have $S(t) \geq 0$, $\forall t \geq 0$. Similarly, we can obtain that $I(t) \geq 0$, $\forall t \geq 0$ and $y(t) \geq 0$, $\forall t \geq 0$. □

In the following, we will prove the locally asymptotical stability of equilibria of system (2).

The equilibria of the system (2) are solutions to the system:

$$D_t^\alpha S(t) = D_t^\alpha I(t) = D_t^\alpha y(t) = 0.$$

System (2) possesses the following biologically feasible equilibria. $E_1(\frac{A}{\mu_1}, 0, 0)$; $E_2(S_2, 0, y_2)$, where $S_2 = \frac{A}{\mu_1}$, $y_2 = \frac{\alpha_2 k_2}{c_2}$; $E_3(S_3, I_3, 0)$, where $S_3 = \frac{\gamma+\mu_2}{\beta}$, $I_3 = \frac{A\beta-\gamma\mu_1-\mu_1\mu_2}{\beta\mu_2}$. Equilibria E_1 and E_2 exist for any parametric value, whereas E_3 exists if $A\beta > \gamma\mu_1 + \mu_1\mu_2$. We now seek the regions of the parameter space for which model system (2) admits a feasible interior equilibrium (equilibria). Any feasible equilibrium must correspond to a positive root I^* of the quadratic equation

$$f(I) = a_1 I^2 + a_2 I + a_3,$$

where

$$\begin{aligned} a_1 &= (c_1\alpha_2 + c_2\mu_2)\beta, \\ a_2 &= c_1k_2\alpha_2\beta + c_2k_1\mu_2\beta + \alpha_2c_1\mu_1 + c_2\gamma\mu_1 + c_2\mu_1\mu_2 - A\beta c_2, \\ a_3 &= \alpha_2c_1k_2\mu_1 + \gamma c_2k_1\mu_1 + c_2k_1\mu_1\mu_2 - A\beta c_2k_1. \end{aligned}$$

for which, additionally,

$$y^* = \frac{\alpha_2}{c_2}(I^* + k_2), S^* = \frac{1}{\beta}(\mu_2 + \gamma + \frac{c_1 y^*}{I^* + k_1}).$$

Let $\Delta = a_2^2 - 4a_1a_3$.

Proposition 1. *If $a_3 < 0$, the system (2) has a unique positive equilibrium.*

Proposition 2. *If $a_2 > 0$ and $a_3 > 0$, there is no positive equilibrium of system (2).*

Proposition 3. *If $a_2 < 0$, $a_3 > 0$ and $\triangle > 0$, there are two positive equilibria of system (2).*

Theorem 2. *E_1 is always unstable.*

Proof. The Jacobian matrix of system (2) evaluated at E_1 is given by

$$J(E_1) = \begin{pmatrix} -\mu_1 & -\frac{A\beta}{\mu_1} & 0 \\ 0 & \frac{A\beta}{\mu_1} - \mu_2 - \gamma & 0 \\ 0 & 0 & \alpha_2 \end{pmatrix}. \quad (4)$$

The eigenvalues can be determined by solving the characteristic equation $det(J(E_1 - \lambda I_3)) = 0$, and they are $\lambda_1 = -\mu_1(< 0)$, $\lambda_2 = \frac{A\beta}{\mu_1} - \mu_2 - \gamma$, and $\lambda_3 = \alpha_2(> 0)$. Note that $|\arg(\lambda_3)| = 0$. Since the eigenvalue λ_3 does not satisfy $|\arg(\lambda_3)| > \frac{\pi}{2}$ for all $\alpha \in (0, 1]$, therefore $E_1(\frac{A}{\mu_1}, 0, 0)$ is always unstable. □

Theorem 3. *If $A\beta c_2k_1 < \mu_1\mu_2c_2k_1 + c_1\alpha_2k_2\mu_1 + \gamma\mu_1c_2k_1$, the equilibrium E_2 is locally asymptotically stable. If $A\beta c_2k_1 > \mu_1\mu_2c_2k_1 + c_1\alpha_2k_2\mu_1 + \gamma\mu_1c_2k_1$, E_2 is unstable.*

Proof. The Jacobian matrix $J(E_2)$ is computed as

$$J(E_2) = \begin{pmatrix} -\mu_1 & -\frac{A\beta}{\mu_1} + \gamma & 0 \\ 0 & \frac{A\beta}{\mu_1} - \mu_2 - \gamma - \frac{c_1k_2\alpha_2}{c_2k_1} & 0 \\ 0 & \frac{\alpha_2^2}{c_2} & -\alpha_2 \end{pmatrix}. \tag{5}$$

The corresponding eigenvalues are $\lambda_1 = -\mu_1(< 0)$, $\lambda_2 = \frac{A\beta}{\mu_1} - \mu_2 - \gamma - \frac{c_1k_2\alpha_2}{c_2k_1}$, and $\lambda_3 = -\alpha_2(> 0)$. Here, two cases arise depending on whether $A\beta c_2k_1 < \mu_1\mu_2c_2k_1 + c_1\alpha_2k_2\mu_1 + \gamma\mu_1c_2k_1$ or $A\beta c_2k_1 > \mu_1\mu_2c_2k_1 + c_1\alpha_2k_2\mu_1 + \gamma\mu_1c_2k_1$.

Case 1. If $A\beta c_2k_1 < \mu_1\mu_2c_2k_1 + c_1\alpha_2k_2\mu_1 + \gamma\mu_1c_2k_1$, then we can see that $|\arg(\lambda_i)| = \pi > \frac{\pi}{2}$, $\alpha \in (0, 1]$, $i = 1, 2, 3$. Therefore, the equilibrium E_1 is locally asymptotically stable.

Case 2. If $A\beta c_2k_1 > \mu_1\mu_2c_2k_1 + c_1\alpha_2k_2\mu_1 + \gamma\mu_1c_2k_1$, then it is easy to see that $|\arg(\lambda_2)| = 0$. In this case, E_2 is unstable. □

Theorem 4. *E_3 is always unstable when it exists.*

Proof. The Jacobian matrix of system (2) evaluated at E_1 is given by

$$J(E_3) = \begin{pmatrix} -\frac{A\beta-\gamma\mu_1}{\mu_2} & \mu_2 & 0 \\ \frac{A\beta-\gamma\mu_1-\mu_1\mu_2}{\mu_2} & 0 & -\frac{c_1(A\beta-\gamma\mu_1-\mu_1\mu_2)}{A\beta+\beta k_1\mu_2-\gamma\mu_1-\mu_1\mu_2} \\ 0 & 0 & \alpha_2 \end{pmatrix}. \tag{6}$$

The characteristic equation of the Jacobian matrix $J(E_3)$ can be expressed as

$$(\lambda - \alpha_2)(\lambda^2 + \frac{A\beta - \gamma\mu_1}{\mu_2}\lambda + A\beta - \gamma\mu_1 - \mu_1\mu_2) = 0.$$

Therefore, one eigenvalue is $\lambda_1 = \alpha_2 > 0$ and $|\arg(\lambda_1)| = 0$. Hence, E_3 is always unstable. □

For the positive equilibrium E^*, the Jacobian matrix is evaluated as

$$J(E^*) = \begin{pmatrix} -\beta I^* - \mu_1 & -\beta S^* + \gamma & 0 \\ \beta I^* & \frac{c_1I^*y^*}{(I^*+k_1)^2} & -\frac{c_1I^*}{I^*+k_1} \\ 0 & \frac{\alpha_2^2}{c_2} & -\alpha_2 \end{pmatrix}. \tag{7}$$

The eigenvalues are the roots of the cubic equation

$$f(\lambda) = \lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0, \tag{8}$$

where

$$A_1 = \alpha_2 + \beta I^* + \mu_1 - \frac{c_1 I^* y^*}{(I^*+k_1)^2},$$
$$A_2 = \frac{c_1\alpha_2^2 I^*}{c_2(I^*+k_1)} - \frac{\alpha_2 c_1 I^* y^*}{(I^*+k_1)^2} + (\beta I^* + \mu_1)(\alpha_2 - \frac{c_1 I^* y^*}{(I^*+k_1)^2}) - \beta I^*(\gamma - \beta S^*),$$
$$A_3 = (\beta I^* + \mu_1)(\frac{c_1\alpha_2^2 I^*}{c_2(I^*+k_1)} - \frac{\alpha_2 c_1 I^* y^*}{(I^*+k_1)^2}) - \beta I^*(\gamma - \beta S^*)\alpha_2.$$

The discriminant $D(f)$ of the cubic polynomial $f(\lambda)$ is

$$D(f) = -\begin{vmatrix} 1 & A_1 & A_2 & A_3 & 0 \\ 0 & 1 & A_1 & A_2 & A_3 \\ 3 & 2A_1 & A_2 & 0 & 0 \\ 0 & 3 & 2A_1 & A_2 & 0 \\ 0 & 0 & 3 & 2A_1 & A_2 \end{vmatrix}. \quad (9)$$

On expansion, one gets $D(f) = 18A_1A_2A_3 + (A_1A_2)^2 - 4A_3A_1^3 - 4A_2^3 - 27A_3^2$.

Now considering the stability conditions in [12], the following theorem can be stated.

Theorem 5. *(1) If $D(f) > 0$, $A_1 > 0$, $A_3 > 0$ and $A_1A_2 - A_3 > 0$, then the interior equilibrium E^* is locally asymptotically stable for $0 < \alpha \leq 1$.*

(2) If $D(f) < 0$, $A_1 \geq 0$, $A_2 \geq 0$, $A_3 > 0$ and $0 < \alpha \leq \frac{2}{3}$, then the interior equilibrium E^* is locally asymptotically stable.

(3) If $D(f) < 0$, $A_1 < 0$, $A_2 < 0$ and $\alpha > \frac{2}{3}$, then the interior equilibrium E^* is unstable.

(4) If $D(f) < 0$, $A_1 > 0$, $A_2 > 0$, $A_1A_2 = A_3$ and $0 < \alpha \leq 1$, then the interior equilibrium E^* is locally asymptotically stable.

4. Numerical simulations

In this section, we present some numerical simulations to illustrate the theoretical results and show the effects of fractional order of the system. We apply the predictor-correctors scheme [19, 20], based on the Adams-Bashforth-Moulton algorithm to solve the numerical solutions of the system (2).

Case 1. The parameters are $A = 15$, $\beta = 0.2$, $\mu_1 = 0.0045$, $\gamma = 0.0032$; $\mu_2 = 0.03$, $c_1 = 0.56$, $k_1 = 2, \alpha_2 = 0.6$, $c_2 = 0.3$, $k_2 = 2.5$, and $\alpha = 1$, 0.95, 0.9 respectively. The system (2) exists one positive equilibrium $E^*(5.957781933, 12.59991544, 30.19983088)$. By calculation, we can obtain $\lambda_{1,2} = -0.2722693531 + 1.015456818i$, $\lambda_3 = -1.580267871$ around E^*. And $|\arg(\lambda_{1,2})| = 1.832759732 > \frac{\alpha\pi}{2}$, $|\arg(\lambda_3)| = \pi > \frac{\alpha\pi}{2}$. Hence, E^* is locally asymptotically stable. See Fig. 1.

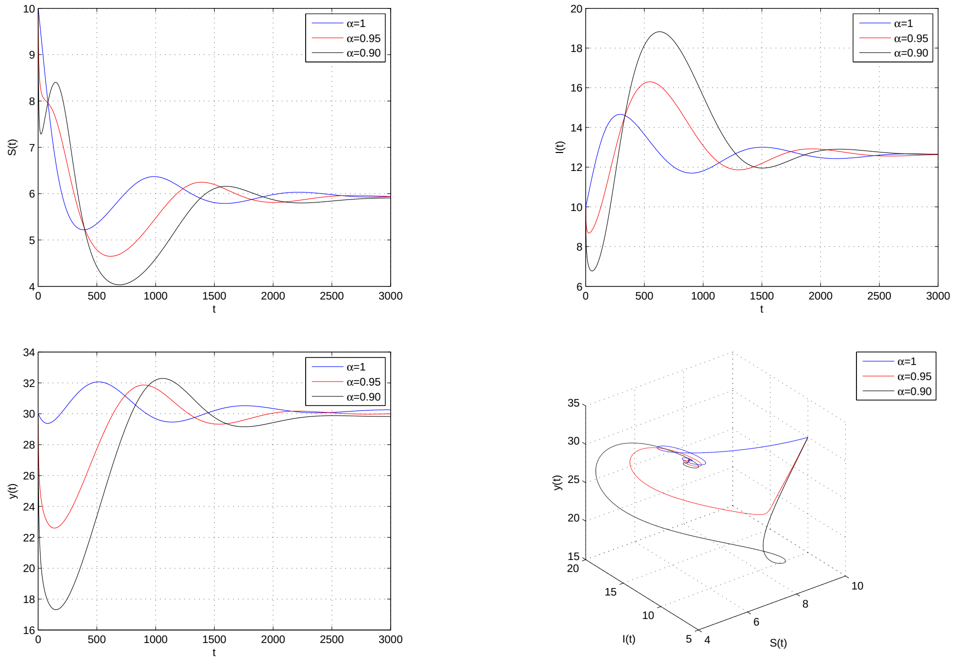

Figure 1: E^* is locally stable. $A = 15$, $\beta = 0.2$, $\mu_1 = 0.0045$, $\gamma = 0.0032$; $\mu_2 = 0.03$, $c_1 = 0.56$, $k_1 = 2$, $\alpha_2 = 0.6$, $c_2 = 0.3$, $k_2 = 2.5$, $\alpha = 1, 0.95, 0.9$

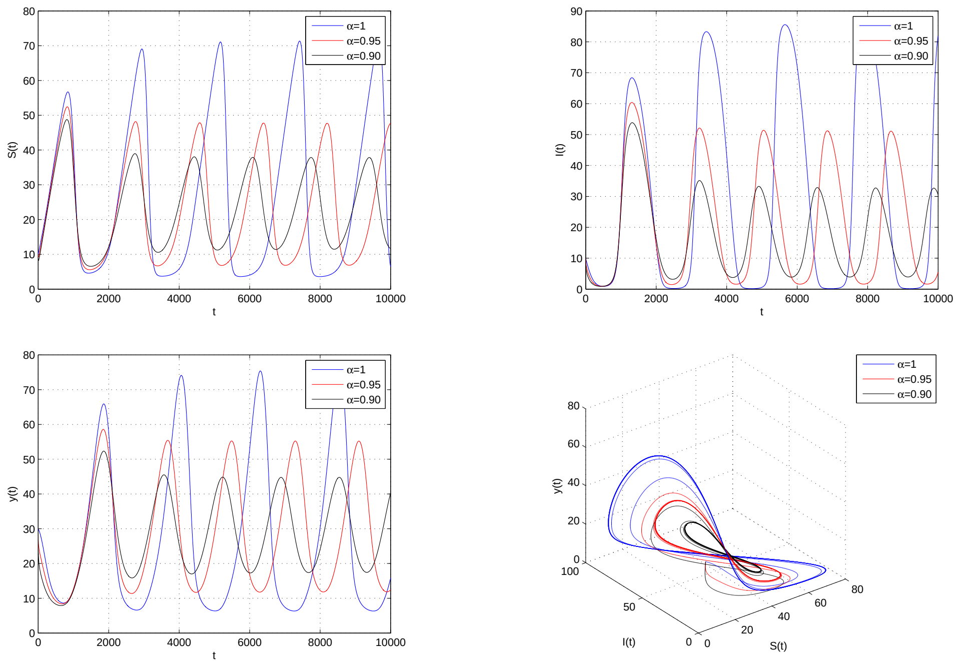

Figure 2: E^* is unstable. $A = 15$, $\beta = 0.05$, $\mu_1 = 0.0045$, $\gamma = 0.0032$; $\mu_2 = 0.03$, $c_1 = 0.56$, $k_1 = 2$, $\alpha_2 = 0.6$, $c_2 = 0.3$, $k_2 = 2.5$, $\alpha = 1, 0.95, 0.9$

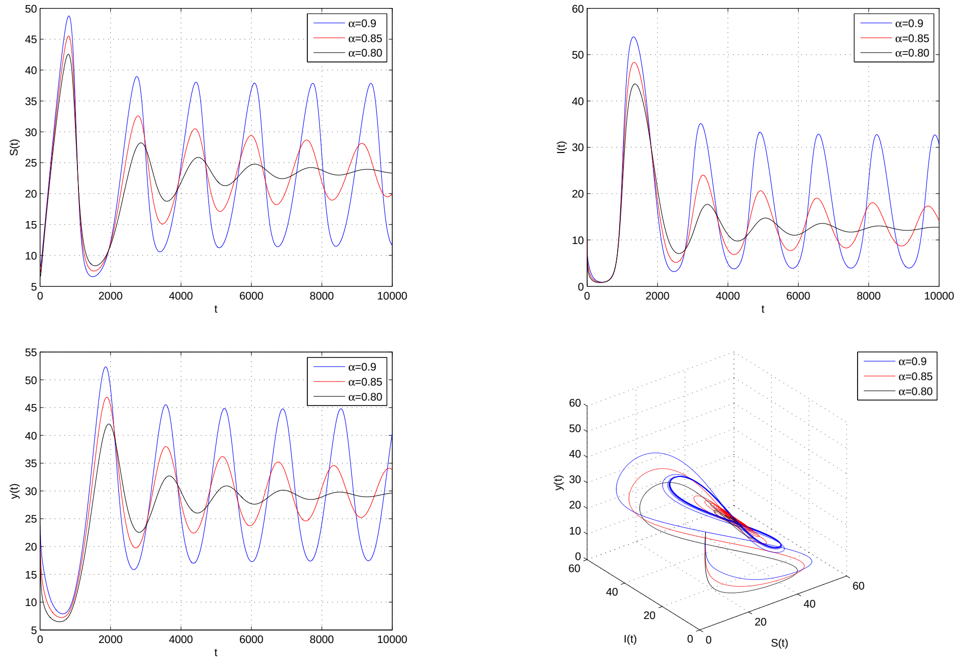

Figure 3: E^* is from unstable to stable as α crease. $A = 15$, $\beta = 0.05$, $\mu_1 = 0.0045$, $\gamma = 0.0032$; $\mu_2 = 0.03$, $c_1 = 0.56$, $k_1 = 2$, $\alpha_2 = 0.6$, $c_2 = 0.3$, $k_2 = 2.5$, $\alpha = 0.9, 0.85, 0.8$

Case 2. The parameters are $A = 15$, $\beta = 0.05$, $\mu_1 = 0.0045$, $\gamma = 0.0032$; $\mu_2 = 0.03$, $c_1 = 0.56$, $k_1 = 2$, $\alpha_2 = 0.6$, $c_2 = 0.3$, $k_2 = 2.5$, and $\alpha = 1, 0.95, 0.9$ respectively. The system (2) exists one positive equilibrium $E^*(7.943844810, 12.59217925, 30.18435851)$. By calculation, we can obtain $\lambda_{1,2} = -0.1536861791 \pm 1.039185550i$, $\lambda_3 = -0.6138627771$ around E^*. And $|\arg(\lambda_{1,2})| = 1.341740339 < \frac{\alpha\pi}{2}$. Hence, E^* is unstable. See Fig. 2.

Case 3. In this case, the parameters are $A = 15$, $\beta = 0.05$, $\mu_1 = 0.0045$, $\gamma = 0.0032$; $\mu_2 = 0.03$, $c_1 = 0.56$, $k_1 = 2$, $\alpha_2 = 0.6$, $c_2 = 0.3$, $k_2 = 2.5$, and $\alpha = 0.9, 0.85, 0.8$ respectively. The system (2) exists one positive equilibrium $E^*(7.943844810, 12.59217925, 30.18435851)$. By calculation, we can get $\lambda_{1,2} = -0.1536861791 \pm 1.039185550i$, $\lambda_3 = -0.6138627771$. When $\alpha = 0.9$, $|\arg(\lambda_{1,2})| = 1.341740339 < \frac{\alpha\pi}{2}$. And E^* is unstable. When $\alpha = 0.85, 0.8$, $|\arg(\lambda_{1,2})| = 1.341740339 > \frac{\alpha\pi}{2}$, $|\arg(\lambda_3)| = \pi > \frac{\alpha\pi}{2}$. And E^* is locally asymptotically stable. From Fig. 3, we conclude that there exists $\alpha^* \in (0, 1]$, E^* is locally asymptotically stable when $\alpha < \alpha^*$ and E^* is unstable when $\alpha > \alpha^*$. See Fig. 3.

5. Conclusion

In this paper, we have studied a fractional order eco-epidemiological model with modified Leslie-Gower Holling-type II schemes. We can get the equilibrium points of the model. And we present the local asymptotic stability of the model. E_1 and E_3 are always unstable. If $A\beta c_2 k_1 < \mu_1\mu_2 c_2 k_1 + c_1\alpha_2 k_2\mu_1 + \gamma\mu_1 c_2 k_1$, the equilibrium E_2 is locally asymptotically stable; If $A\beta c_2 k_1 > \mu_1\mu_2 c_2 k_1 + c_1\alpha_2 k_2\mu_1 + \gamma\mu_1 c_2 k_1$, E_2 is unstable. And the positive equilibrium (equilibria) is locally asymptotically stable under some conditions. Some numerical simulations are provided to illustrate the theoretical results and the effects of fractional order of the system. Numerical simulations indicate fractional order α is a factor which affects the behavior of solutions. There exists $\alpha^* > 0$ such that if $\alpha \in [0, \alpha^*)$ the equilibrium point is asymptotically stable. If $\alpha^* < \alpha$, then the equilibrium point becomes unstable. That is to say, the system (2) undergoes a Hopf bifurcation at the equilibrium E^* when the fractional order α passes through the critical value α^*.

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