http://mmg.tversu.ru ISSN 2311-1275

MMG *Mathematical Modelling and Geometry*

Volume 6, No 3, pp. 9 – 21 (2018) **Full text**

Bound orbits near the throats of phantom scalar field wormholes

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Abstract. We consider asymptotically flat, static, traversable wormholes supported by a gravitating minimally coupled phantom scalar field with an arbitrary self-interaction potential. It turns out that the main features of bound orbits in wormhole spacetimes are radically different from those in static black hole spacetimes. First, on the throat or near it, there necessarily exists a stable circular orbit in which any test particle has zero angular momentum; this marginal orbit is a degenerate analogue of the innermost stable circular orbit near black holes. Thus, particles of matter resting on these orbits or slowly moving near them can form a thin spherical shell consisting of gas, dust, or fluid. Second, the distance to the throat from an orbit of a test particle with a sufficiently small specific angular momentum can, unlike for the orbits around vacuum black holes, reaches its minimum and maximum values arbitrarily many times (multiple precession — periapsis precession with a very large deficit angle) during one full revolution around the centre.

Keywords: wormhole, phantom scalar field, marginal bound orbits

MSC numbers: 83C10, 83C57

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