



On the join products of two special graphs on five vertices with the path and the cycle

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Abstract. The investigation on the crossing numbers of graphs is very difficult problem provided that an computing of the crossing number of a given graph in general is NP-complete problem. The problem of reducing the number of crossings in the graph is studied not only in the graph theory, but also by computer scientists. The exact values of the crossing numbers are known only for some graphs or some families of graphs. In the paper, we extend known results concerning crossing numbers for join products of two graphs of order five with the path P_n and the cycle C_n on n vertices. The methods used in the paper are new, and they are based on combinatorial properties of cyclic permutations.

Keywords: graph, drawing, crossing number, join product, cyclic permutation

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