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Numerical modeling of color perception of optical radiation

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Abstract. The concept of color is closely related to how a person perceives light. It can be said that the perception of light is formed by the human brain as a result of the analysis of the light flux falling on the retina of the eye. Color is the result of the interaction of the light flux and the observer (or recording device). The basis of the mathematical description of color is the experimentally established fact that any color can be represented as a mixture of certain quantities of three or more linearly independent colors. The paper gives a description of four color spaces used in the analysis and design of light reflecting, light-transmitting and light-emitting devices. Such devices include liquid crystal displays (LCD), solar panels, reflective and sunproof glasses, as well as polarized glasses. Numerical calculations (carried out using the MorphoVision software package, developed by the authors of the paper) are presented for the color coordinates of thin-film multilayer structures consisting of several layers of isotropic and anisotropic materials of different thicknesses.

Keywords: color space, color coordinates, reflectance characteristics, emission spectrum, calculation of color coordinates of a multilayer structure

MSC numbers: 68N30 ,

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1. Introduction

Color of an object is the result of interaction between light, object, and observer. The light transmitted through or reflected from the object of observation enters the eye of an observer, and the observer percepts the light transformed by the object as possessing a certain color. For the correct measurement and reproduction of color characteristics by means of various devices, it is necessary to use standard models for the description of light sources and the character of color perception by an observer. A commonly accepted model of color perception by human eye is the so-called tristimulus model (RGB model), which is based on the assumption that the human eye distinguishes three principal colors — red (R), green (G), and blue (B), while all other colors are obtained as a result of additive processing of the obtained visual information in the brain.

The problem of correct determination of the color coordinates is very complicated and, despite many years of investigation, is still far from satisfactory solution [1], although there are commonly accepted and employed standard photometric measurement procedures developed by *Commission Internationale de l'Eclairage* (CIE) [2].

The relative spectral responsivity of the human eye for numerical calculations was first defined by CIE in 1924 [2]. It is called the *spectral luminous efficiency function for photopic vision*, or the $V(\lambda)$ function. This function is defined in the wavelength range from 360 to 830 nm and normalized to one of the peaks, namely, to that at 555 nm (Fig. 1). This model gained wide acceptance. The tabulated values of the function at 1 nm increments are available in references [2]. In most cases, the region from 380 to 780 nm is used for calculation with negligible errors because the $V(\lambda)$ function falls below 10^{-4} outside this region. Thus, a photo detector having a spectral responsivity matched to the $V(\lambda)$ function can perform the role of the human eyes in photometry.

It should be noted that the $V(\lambda)$ function is based on the CIE *standard photometric observer for photopic vision*, which assumes additivity of sensation and a $2°$ field of view at relatively high luminance levels (higher than ~ 1 cd/m^2). The human vision at this level is called *photopic* vision.

2. Calculating colors in LCDs

From the fundamental point of view, any source of color light can be described by its spectral distribution. This can be easily obtained experimentally by using a spectrometer such as a diffraction grating or a prism. The human eye, however, does not function like a spectrometer. It is generally believed that human eyes contain three different pigments (say, R, G, and B), which receive the light with different absorption spectra, so that R pigments absorb strongly in the red region, G pigments absorb strongly in the green, and B pigments absorb strongly in the blue. Thus, when a beam of light is directed into the eyes, we perceive a color that is the result of mixing of the amount of absorption received by the three sets of

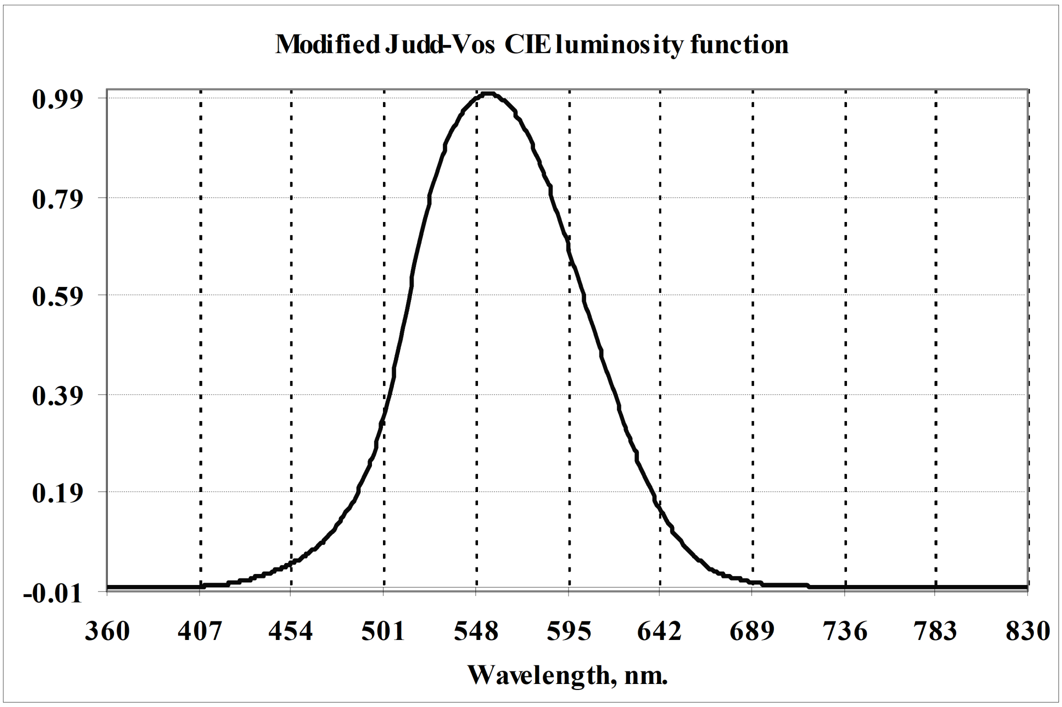


Figure 1: Modified CIE 2° photopic luminosity function.

pigments. As a result of the mechanism of color vision of human eyes, different spectral distribution can produce the same color. In addition, any color can be made from three different colors (e.g., red, green, and blue). These fundamental laws of colors can be described mathematically, see [3].

In most color LCDs (**liquid-crystal displays**), each pixel of information consists of three sub pixels with color filters. The color perceived by our eyes is a mixture (sum) of three colors in the pixel. Denoting the intensity transmission coefficients for the sub pixels t_1, t_2, t_3, and the transmission spectra of the corresponding color filters $f_1(\lambda)$, $f_2(\lambda)$, $f_3(\lambda)$, we can define the effective transmission spectrum of the pixel as

$$T(\lambda) = t_1 f_1(\lambda) + t_2 f_2(\lambda) + t_3 f_3(\lambda).$$

Here, we assume that t_1, t_2, t_3 are insensitive to variations of the light wavelength. Strictly speaking, t_1, t_2, t_3 are the functions of the wavelength. However, the spectral dependence of $T(\lambda)$ is dominated by the transmission of the color filters $f_1(\lambda)$, $f_2(\lambda)$, and $f_3(\lambda)$. In color LCDs, t_1, t_2, t_3 can be independently controlled by applying different voltages to each sub pixel. Once $T(\lambda)$ is obtained, it is possible to calculate the chromaticity coordinates of the color transmitted by the pixel. In what follows, we will consider the calculation of the chromaticity coordinates and the mixing of colors.

In order to calculate the chromaticity coordinates (x, y), we must first obtain the *tristimulus values* (X, Y, Z) by using the following formulas [3, 4]:

$$X = k \int S(\lambda)T(\lambda)\bar{x}(\lambda)\,\mathrm{d}\lambda,$$
$$Y = k \int S(\lambda)T(\lambda)\bar{y}(\lambda)\,\mathrm{d}\lambda,$$
$$Z = k \int S(\lambda)T(\lambda)\bar{z}(\lambda)\,\mathrm{d}\lambda.$$

where $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ are the *color matching functions*, $S(\lambda)$ is the source spectrum (backlight illuminant), $T(\lambda)$ is the transmission spectrum of the information pixel; and k is the normalization factor defined as

$$k = \frac{100}{\int S(\lambda)\bar{y}(\lambda)\,\mathrm{d}\lambda}.$$

This definition of k makes the Y tristimulus value equal to 100 for a perfect transmission system with $T(\lambda) = 1$ for all λ. The color matching functions of CIE 1931 *Standard Colorimetric Observer* are denoted $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ for 1–4° viewing angles. Spectral tristimulus values are plotted as functions of the wavelength in Fig. 2.

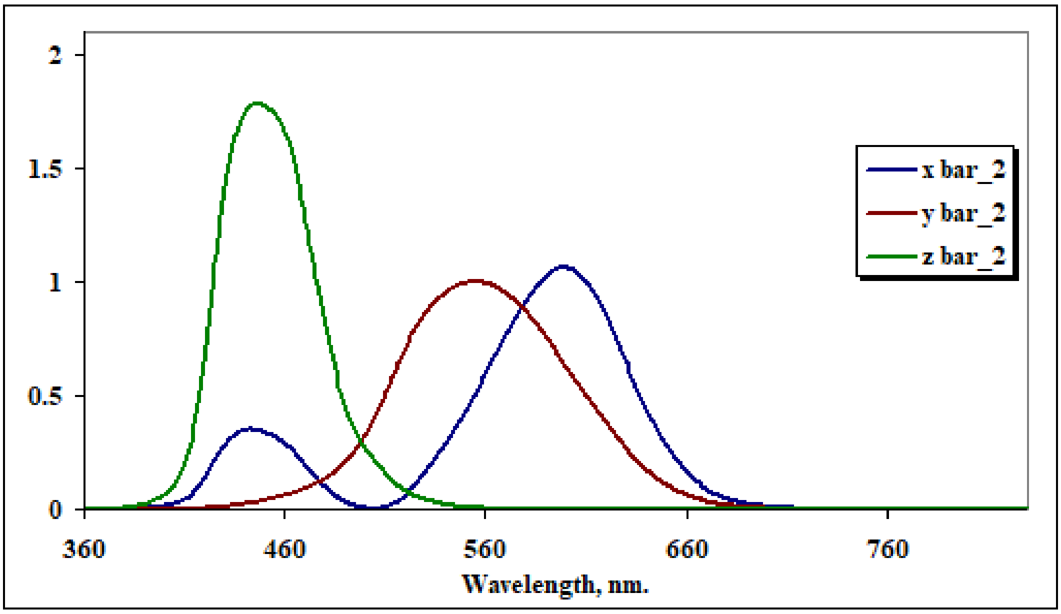


Figure 2: Spectral Luminous Efficiency Function for Photopic Vision.

The chromaticity coordinates of any given color are defined as follows:

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}.$$

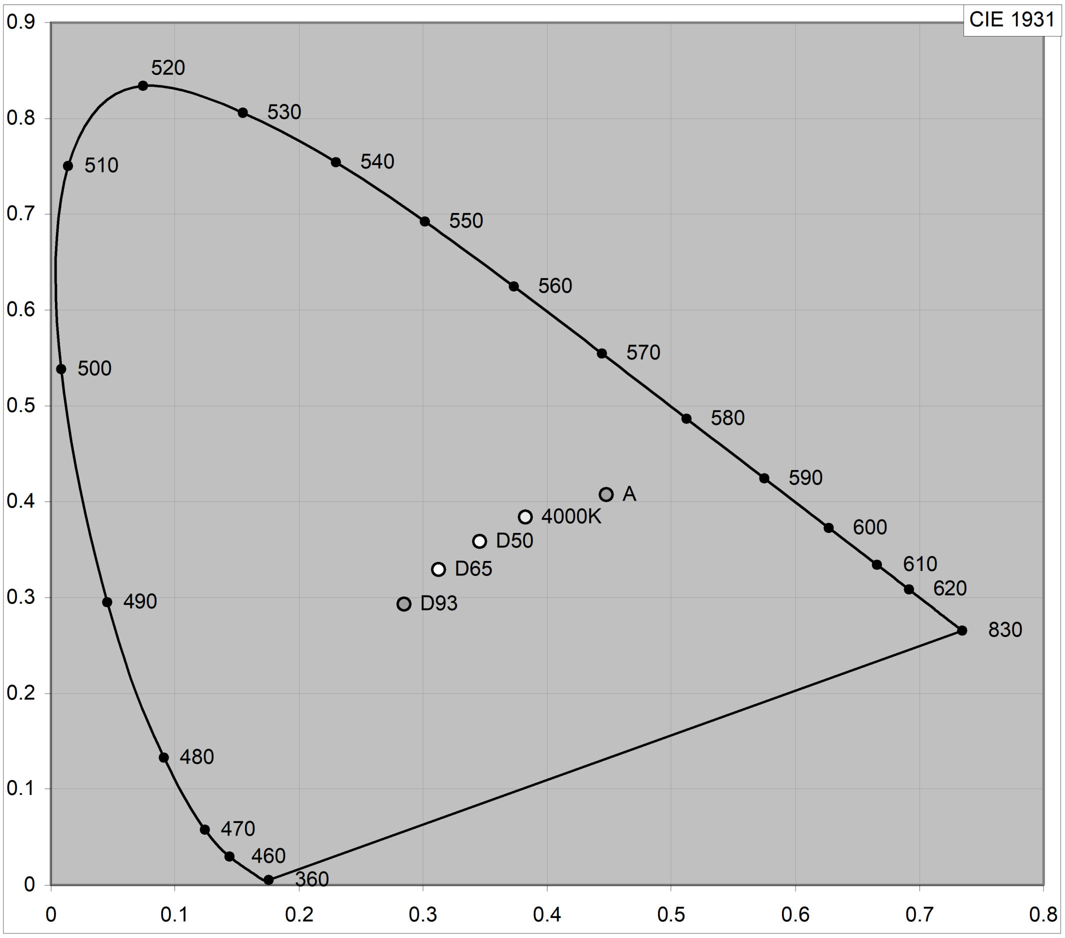


Figure 3: CIE 1931 (x, y) 2° chromaticity diagram showing the spectrum locus of monochromatic stimuli and White points for different illuminants.

Each color specification may be represented by a point (x, y) in the chromaticity coordinate system. Figure 3 shows a curve (from 360 to 830 nm) constructed by connecting all points of the chromaticity coordinates of monochromatic light, and the purple line, which is straight line between beginning and ending points.

Some white-point coordinates for different illuminants, which we used for testing algorithms, are given in Table 1.

Remark. To calculate equienergy white light (white-point), consider an artificial source of white light with $S(\lambda) = \text{const}$. Using $T(\lambda) = 1$, tabulated tristimulus values of the color matching functions for 2 degree viewing angles and equations for (X, Y, Z), we can show that tristimulus values $X = Y = Z$ are the same and that the chromaticity coordinates are $x = 0.3333$, $y = 0.3333$ (equienergy white).

Table 1: Some white-point coordinates calculated for 360–830 nm wavelength range

Illuminant name	Provided by	x	y
CIE Std illuminant A	Tungsten lamp, normal incandescent	0.4476	0.4074
D5000 or D50	Bright tungsten (low voltage bulb at serious over voltage)	0.3457	0.3585
CIE Std illuminant B	Direct sunlight	0.3484	0.3516
D5500 or D55	Cloudy daylight	0.3324	0.3474
CIE Std illuminant E	Normalized reference source	1/3	1/3
D6500 or D65	Daylight	0.312713	0.329016
CIE Std illuminant C	Average Sunlight	0.3101	0.3162
D7500 or D75		0.299	0.3149
D9300	Default of some old or low-quality CRT monitors, very bad for imaging	0.2848	0.2932

3. Color spaces

Color is the brain reaction to a specific visual stimulus. Although we can precisely describe color by measuring its spectral power distribution (the intensity of the visible electromagnetic radiation at many discrete wavelengths) this leads to a large degree of redundancy. The reason for this redundancy is that the eye retina samples color using only three broad bands, roughly corresponding to red, green and blue light. The signals from these color sensitive cells (cones), together with those from the rods (sensitive to intensity only), are combined in the brain to give several different "sensations" of the color. These sensations have been defined by CIE:

- ***Brightness***: the human sensation by which an area exhibits more or less light.
- ***Hue***: the human sensation according to which an area appears to be similar to one, or to proportions of two, of the perceived colors red, yellow, green and blue.
- ***Colorfulness***: the human sensation according to which an area appears to exhibit more or less of its hue.
- ***Lightness***: the sensation of an area brightness relative to a reference white in the scene.
-

- ***Chroma***: the colorfulness of an area relative to the brightness of a reference white.

- ***Saturation***: the colorfulness of an area relative to its brightness.

The trichromatic theory describes the way in which three separate lights, red, green and blue, can match any visible color — based on the eye's use of three color sensitive sensors. This is the basis on which photography and printing operate, using three different colored dyes to reproduce color in a scene. It is also the way that most computer color spaces operate, using three parameters to define a color.

A color space is a method by which we can specify, create and visualize color. As humans, we may define a color by its attributes of brightness, hue and colorfulness. A computer may describe a color using the amounts of red, green and blue phosphor emission required to match a color. A printing press may produce a specific color in terms of the reflectance and absorbance of cyan, magenta, yellow and black inks on the printing paper.

A color is thus usually specified using three coordinates, or parameters. These parameters describe the position of the color within the color space being used. They do not tell us what the color is, that depends on what color space is being used.

Different color spaces are better for different applications, for example some equipment has limiting factors that dictate the size and type of color space that can be used.

Some color spaces are perceptually linear, that is, a 10-unit change in stimulus will produce the same change in perception wherever it is applied. Many color spaces, particularly in computer graphics, are not linear in this way.

Some color spaces are intuitive to use, so that it is easy for the user to navigate within them, and creating desired colors is relatively easy. Other spaces are confusing for the user with parameters with abstract relationships to the perceived color.

Finally, some color spaces are tied to a specific piece of equipment (i.e. are device dependent) while others are equally valid on whatever device they are used.

A color can be described as a mixture of three other colors or "Tristimulus". Typically RGB for CRT based systems (TV, computer) or XYZ (fundamental measurements). The amounts of each stimulus define the color. However, it is frequently useful to separate the color definition into "luminance" and "chromaticity". Lower case is always used to signify chromaticity coordinates; upper case always signifies tristimulus values (or amounts of the primaries). Chromaticity coordinates can be plotted on a two-dimensional diagram that defines all the visible colors; luminance is normal to that diagram.

- ***CIE XYZ (1931)*** The CIE XYZ (1931) system is at the root of all colorimetry. It is defined such that all visible colors can be determined using only positive values, and, the Y value is luminance. Consequently, the colors of the (X, Y, Z) primaries themselves are not visible. The chromaticity diagram is highly nonlinear, in that a vector of unit magnitude representing the difference between two chromaticities is not uniformly visible. A color defined in this system is referred to as Yxy. A third coordinate, z can also be defined but it is redundant since $x + y + z = 1$ for all colors. The coordinates x and y are defined as:

$$x = \frac{X}{X + Y + Z}, \quad x = \frac{Y}{X + Y + Z}.$$

- ***CIE YUV (1960)*** This is a linear transformation of Yxy, in an attempt to produce a chromaticity diagrams in which a vector of unit magnitude (difference between two points representing two colors) is equally visible at all colors. Y is unchanged from XYZ or Yxy. Difference in non-uniformity is reduced considerably, but not enough. A third co-ordinate, w, can also be defined but it is redundant. The color coordinates u and v are defined as:

$$u = \frac{2x}{6y - x + 1.5}, \quad v = \frac{3y}{6y - x + 1.5}.$$

- ***CIE YU'V'*** This is another linear transformation of Yxy. Y remains unchanged. Difference in non-uniformity is further reduced, but still not enough. Again, a third co-ordinate, w', can be defined, but it is redundant. The color coordinates u' and v' are defined as:

$$u' = \frac{2x}{6y - x + 1.5}, \quad v' = 1.5v = \frac{4.5y}{6y - x + 1.5}.$$

- ***The Hunter Lab color space*** The Hunter Lab color scale is more visually uniform than the XYZ color scale. In a uniform color scale, the differences between points plotted in the color space correspond to visual differences between the colors plotted. The color coordinates are defined as:

$$(H)L = 10\sqrt{Y},$$
$$(H)a = 1.75(1.02 \cdot X - Y)/\sqrt{Y},$$
$$(H)b = 7(Y - 0.847 \cdot Z)/\sqrt{Y}.$$

The 3D diagram of the Hunter Lab color space is presented in Fig. 4. The L axis runs from top to bottom. The maximum for L is 100, which would be a perfect reflecting diffuser. The minimum for L would be zero, which would be black. The a and b axes in the plane perpendicular to the L axis have no specific numerical limits. Positive a is red. Negative a is green. Positive b is yellow. Negative b is blue.

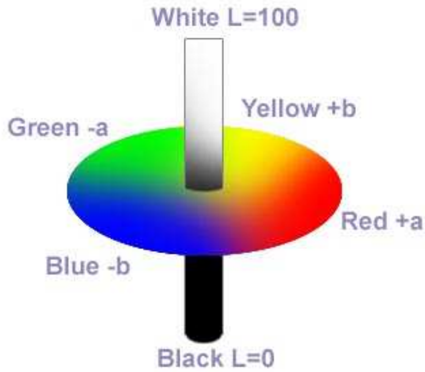


Figure 4: The Hunter Lab color space.

4. Standard illuminants

The tristimulus XYZ values can be calculated by using any illuminant spectral distribution $P(\lambda)$. The attained tristimulus values and chromaticity coordinates depend both of the reflectance characteristics of the material and the emission spectrum of the illuminant. Therefore, to be able to compare colorimetric characteristics of materials, it was necessary to agree on a number of representative spectral distributions. This has been done by defining some spectral power distributions, called *illuminants*. They are described only with the tabulated values of the spectral power distribution and are used to calculate the tristimulus values and chromaticity coordinates. In order to perform visual observations, reproducible sources are needed. These are called *standard sources*.

In 1931, CIE standardized three illuminants and standard sources (in these definitions, we give the temperature values as correct in the International Practical Temperature Scale of 1968):

- ***CIE Standard Illuminant A***: An illuminant having the same relative spectral power distribution as a Planckian radiator at a temperature of 2856 K.

- ***CIE Standard Illuminant B***: An illuminant having the relative spectral power distribution near to that of direct sunlight. (This illuminant is now obsolete.)

- ***CIE Standard Illuminant C***: An illuminant representing average daylight with a correlated color temperature of about 6800 K. (This illuminant is now obsolete.)

A series of daylight illuminants were suggested in 1963 and standardized in 1967. The relative spectral power distribution of these illuminants can be calculated in the following way.

First the correlated color temperature of the daylight (D) has to be chosen: T_C. With this value of T_C one calculates the 1931 x_D chromaticity of the daylight:

- for correlated color temperatures from approximately 4000 K to 7000 K:

$$x_D = -4.6070\frac{10^9}{T_C^3} + 2.9678\frac{10^6}{T_C^2} + 0.09911\frac{10^3}{T_C} + 0.244063$$

- for correlated color temperatures from 7000 K to approximately 25 000 K:

$$x_D = -2.0064\frac{10^9}{T_C^3} + 1.9018\frac{10^6}{T_C^2} + 0.24748\frac{10^3}{T_C} + 0.237040$$

With known x_D value, the y_D value can be obtained, and with the help of both, the spectral power distribution of the Daylight illuminant can be calculated:

- $y_D = -3.000x_D^2 + 2.870x_D - 0.275$

and

- $S(\lambda) = S_0(\lambda) + M_1S_1(\lambda) + M_2S_2(\lambda)$,
  where $S_0(\lambda)$, $S_1(\lambda)$, $S_2(\lambda)$ are the functions of the wavelength λ, usually given in the tabulated form [2]; and M_1, M_2 are factors related to the chromaticity coordinates x_D and y_D as follows:

$$M_1 = \frac{-1.3515 - 1.7703x_D + 5.9114y_D}{0.0241 + 0.2562x_D - 0.7341y_D}$$

$$M_2 = \frac{0.0300 - 31.4424x_D + 30.0717y_D}{0.0241 + 0.2562x_D - 0.7341y_D}$$

Based on this calculation procedure, the relative spectral power distribution of any daylight illumination between 4000 K and 25 000 K can be calculated.

However, for practical reasons, it has been recommended to restrict the daylight illuminants used to a few illuminants.

The CIE standardized the illuminants A and D65 (correlated color temperature of approximately 6504 K) as its primary colorimetric illuminants and one of them should be used whenever possible. Figures 5 and 6 from [2, 5] shows the relative spectral power distribution of CIE Standard Illuminants A and D65.

Spectral power distribution refers to the wavelengths that make up the light emitted from a source or illuminant at a particular color temperature. Those with cooler color temperatures emit the longer wavelengths (red to yellow) in stronger amounts than the shorter wavelengths (blue to violet). Hotter blackbodies emit all wavelengths in more equal distributions, though tending to be slightly stronger in the blue to violet wavelengths.

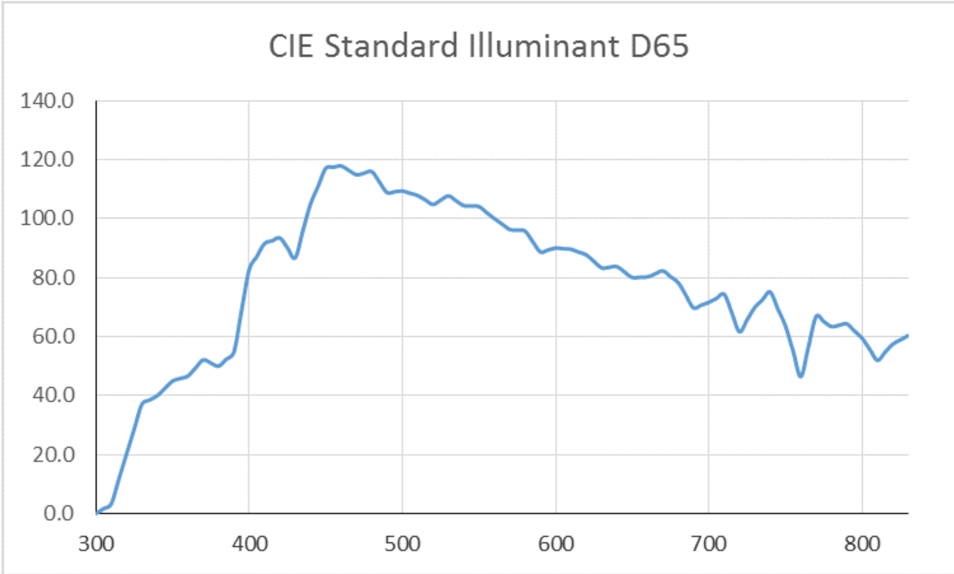


Figure 5: Relative spectral power distribution of CIE Standard Illuminant D65.

Many practical lighting situations differ considerably from the standard incandescent light (Illuminant A) and daylight (D65) situations. Problems are encountered also if visual estimates are needed and no practical source is available.

The spectra of twelve representative fluorescent lamps have been published by CIE for use when colors of samples have to be judged under different indoor lighting situations. One of them compared with the average daylight one can see on the right of Figure 7, which shows the relative spectral power distribution of Normal Florescent lamp. Three of them are most popular: F2 is a standard fluorescent lamp of medium color temperature, F7 is one of daylight color temperature and broad band spectrum, and while F11 comes nearest to modern so-called "tree band" fluorescent lamps — such phosphors are now widely used in the modern small diameter compact fluorescent lamps.

One can compare the spectral power distributions of average daylight and a normal florescent light source at the Figure 7.

The florescent source is relatively low in terms of relative power as compared to CIE Source A (a tungsten-filament bulb) and average daylight and its relative power spikes sharply at certain wavelengths. These spikes are also typical of gas-discharge lamps.

5. Calculating the color coordinates of multilayer structures using the MorphoVision program

When calculating the color coordinates of multilayer structures using the MorphoVision software, it is possible to obtain (parameters of) the intensities of the trans-

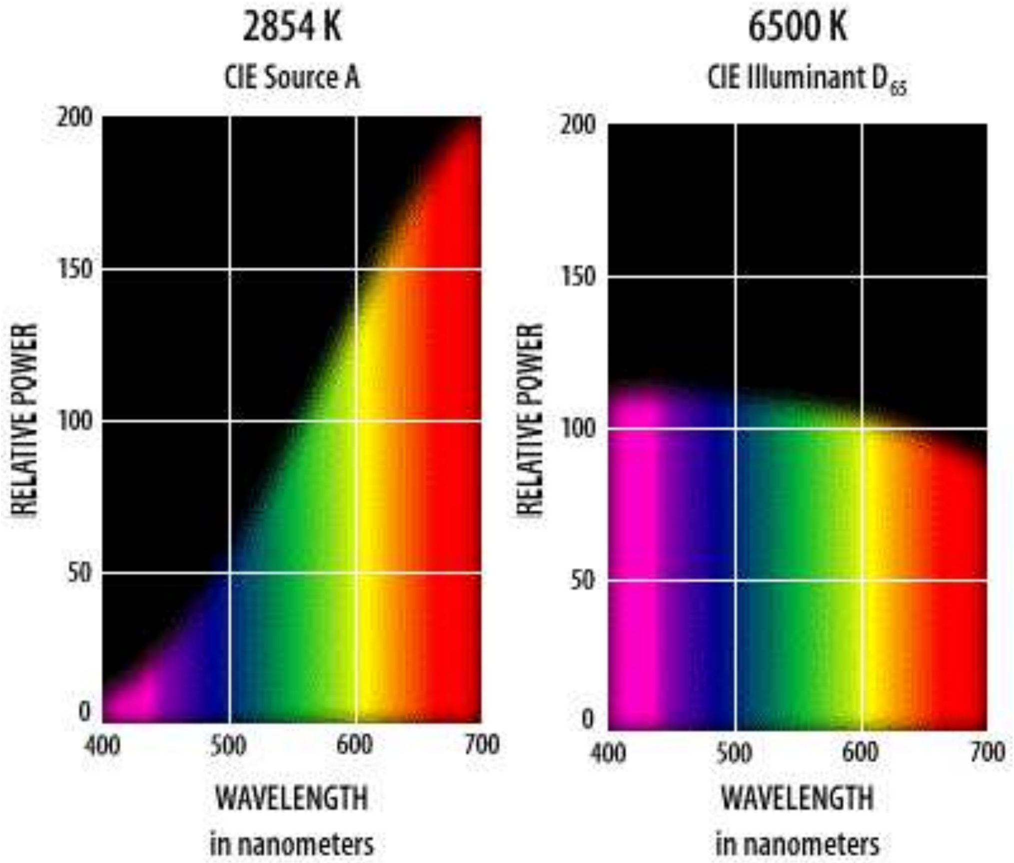


Figure 6: Relative spectral power distribution of CIE Standard Illuminants A and D65.

mitted and reflected light. As a result, it allows to represent the color coordinates in different color systems, depending on the needs of the user. Thus, for example, in the manufacture of LCD displays the most popular system is Hunter Lab color space. The Hunter L,a,b color scale is more visually uniform than the XYZ color scale. In a uniform color scale the differences between points plotted in the color space correspond to visual differences between colors plotted. The CIELAB scale, although designed specifically to be more uniform, is still a bit over-expanded in the yellow region.

Calculated Color parameters.

- x, y — color coordinates in CIE XYZ (1931) system for the transmitted (T) and reflected (R) light;
- u, v — color coordinates in CIE XUV (1960) system for the transmitted and reflected light;
- u', v' — color coordinates in CIE XU'V' system for the transmitted and reflected light;

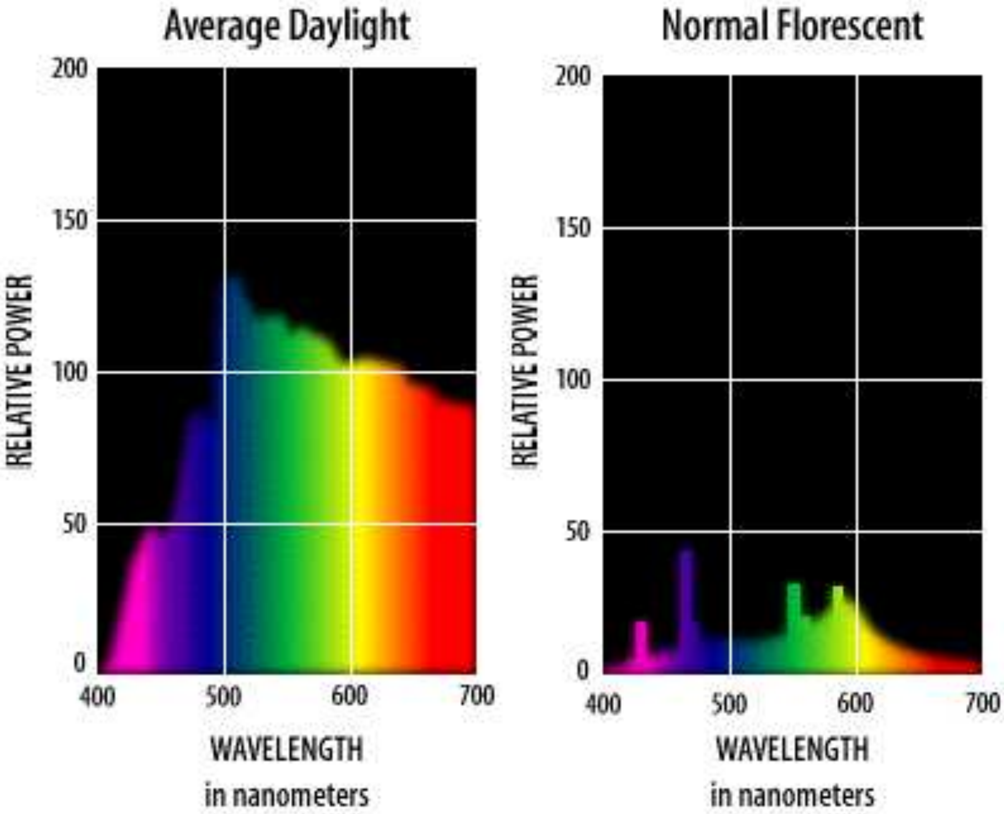


Figure 7: Relative spectral power distribution of Normal fluorescent lamp of 4230 K (F2).

- a, b Hunter — color coordinates in Hunter Lab color scale for the transmitted and reflected light;
- Visual representation of the color coordinates of a given point on a chromaticity diagram for the transmitted and reflected light. Each of the coordinate systems (x, y), (u, v), (u', v') is represented by its own chromaticity diagram. In addition, it is possible to select the color systems of Society of Motion Picture and Television Engineers (SMPTE) or National Television System Committee (NTSC).

It should be noted that color coordinates are calculated for a particular specified light source.

Let's calculate the color coordinates of a multilayer structure consisting of several layers of isotropic and anisotropic materials with different thicknesses. As an example, consider a multilayer structure applied to a glass substrate. The applied structure consists of two repeating packages, each of which contains isotropic and two anisotropic layers of materials with predetermined dielectric properties and thickness. All these parameters are set in the dialog box of the program, the result for a specific example is shown in Fig. 8.

#/Stack	Type, qnty	d (nm)	n_a (...	k_a (...	n_b (ne)	k_b (ke)	n_c	k_c	Theta
0/0	Air_sp		1.0	0.0	1.0	0.0	1.0	0.0	0.0
1/1	Iso	1000.00	1.450...	5.00E-7	1.45000	5.00E-7	1.45000	5.00E-7	0.0
2/1	2ax	400.00	1.500...	0.0	2.80000	0.0	2.45000	5.00E-7	0.0
3/1	2ax	400.00	1.500...	0.0	0.80000	5.00E-1	0.45000	5.00E-3	0.0
#/1	**2 time...**								
4/2	Sub	100000...	1.520...	0.0	1.52000	0.0	1.52000	0.0	0.0
#/2	**1 time...**								
5/0	Air_sp		1.0	0.0	1.0	0.0	1.0	0.0	0.0

Figure 8: Parameters of the test multilayer structure for calculating color parameters.

Calculated Transmittance and Reflectance are calculated first, the result of the calculation is shown in Fig. 9.

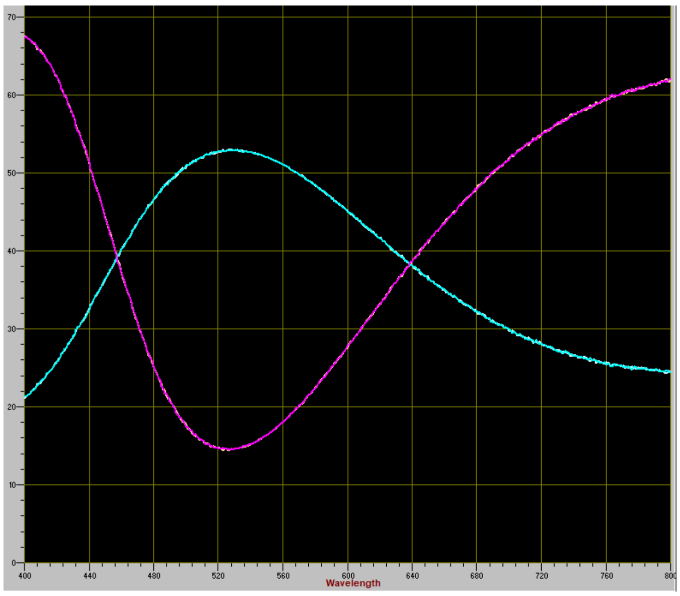
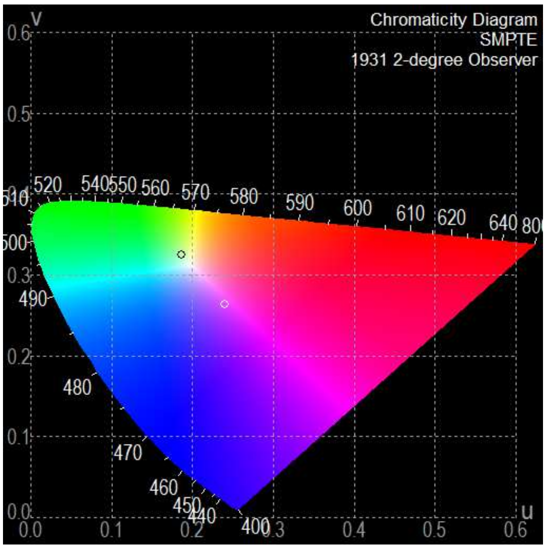


Figure 9: Calculated Transmittance and Reflectance as a function of wavelength in nm.

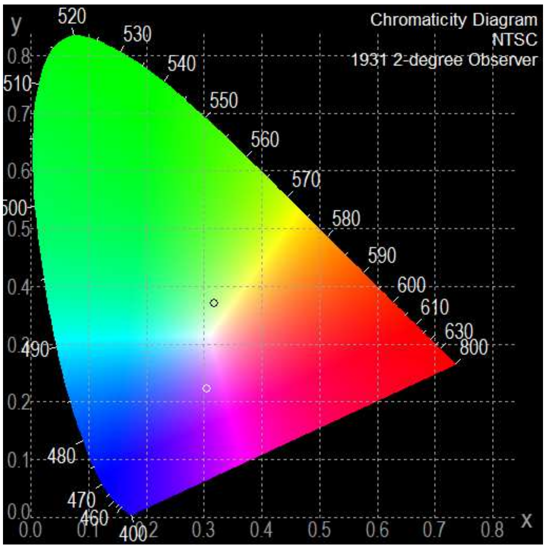
The results of calculating the color coordinates from the found transmission and reflection of this multilayer structure when interacting with the standard D65 light source in different color spaces are shown in Fig. 10.

6. Conclusion

The properties of color vision are taken into account in colorimetry, based on the results of experiments with color mixing. In such experiments, visual equalization



SMPTE coordinates of transmitted and reflected light



NTSC coordinates of transmitted and reflected light

Figure 10: Defined colors in SMPTE and NTSC systems for standard D65 source light.

of the perceived color of optical radiation with mixtures of three or more primary colors is performed.

We have developed the software complex MorphoVision, which calculates the optical properties of multilayer anisotropic thin-film coatings, including analysis of the spectral composition of transmitted and reflected polarized light. In the course of the analysis, software-based mathematical models of colorimetry were used.

The results of numerical calculations presented in the work demonstrate the possibility of representing the optical radiation under study in different color spaces. On the other hand, our numerical results allow us to compare the capabilities of the MorphoVision software suite with other well-known software packages of such companies as StellarNet Inc. [6], Avantes [7], Gooch & Housego [8] and many others.

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