



## Approximate Formulas for Mathematical Expectations of Functionals of Random Processes Defined by Ito-Levy Multiple Integral Expansion

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**Abstract.** An approximate formula is constructed for calculating mathematical expectations of nonlinear functionals of random processes defined by chaotic expansions in terms of multiple Ito-Levy integrals. We consider the possibility of applying the formula to the calculation of mathematical expectation of functionals corresponding to the solution of one kind of Skorokhod equations on a Wiener process. Test examples of computations using the constructed formula for particular cases of Levy process are presented. The elaborated method gives a new useful tool for numerical integration with a required accuracy of stochastic differential equations which are reduced to the Skorokhod class of equations.

**Keywords:** functionals of Levy processes, Ito-Levy multiple integral expansion, mathematical expectations of process functionals, approximate formulae.

**MSC numbers:** 65C30, 60H35

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