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Circular orbits around static self-gravitating scalar field configurations

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Abstract. This article deals with innermost stable circular orbits (ISCOs) of neutral test particles around static, self-gravitating, spherically symmetric configurations that are formed and supported in equilibrium by a self-interacting real scalar field minimally coupled to gravity. For such objects, the spacetime metric can be expressed as the result of actions of some nonlinear integral operators (quadratures) on the scalar field. The quadratures do not depend on the form of the scalar field self-interaction potential. This feature enables us to classify the possible ISCOs, and to study the properties of various kinds of circular orbits without having to solve the Einstein-Klein-Gordon equations. It turns out that there exist two kinds of ISCOs, say the first and second kind, which are characterized, respectively, by a nonzero and zero specific angular momentum J of a test particle. We show that in the case of a classical scalar field, black holes have ISCOs only of the first kind. Naked singularities of general type have ISCOs only of the second kind, while some fine-tuning naked singularities and regular configurations have stable circular orbits of any positive radius. Black holes supported by phantom scalar fields can have ISCOs of both the kind depending on the Schwarzschild mass: for any one-parameter family of black holes parameterized by the mass, there exists a value $m_c > 0$ such that ISCOs of the first and second kind are in the intervals $m_c < m < \infty$ $(J > 0)$ and $0 < m \leqslant m_c$ $(J = 0)$, respectively. The orbital radius r_{ISCO} reaches its minimum value at $m = m_c$ and goes to infinity as m goes to zero or infinity.

Keywords: innermost stable circular orbit, scalar hair, black hole, wormhole, naked singularity, topological geon
MSC numbers: 83C10, 83C57

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