http://mmg.tversu.ru ISSN 2311-1275

Mathematical Modelling and Geometry

Volume 3, No 1, p. 43 – 63 (2015)

Waveguide modes of a planar optical waveguide

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Abstract. In a planar regular optical waveguide, propagation of polarized monochromatic electromagnetic radiation obeys a law following from the Maxwell equations. The Maxwell equations in Cartesian coordinates associated with the waveguide geometry can be written as the two independent systems of equations:

$$E_x = \frac{\beta}{\varepsilon} H_y,\ \frac{dE_z}{dx} = \frac{ik_0}{\varepsilon}\left(\varepsilon\mu - \beta^2\right) H_y,\ \frac{dH_y}{dx} = ik_0 \varepsilon E_z,$$

$$H_x = -\frac{\beta}{\mu} E_y,\ \frac{dH_z}{dx} = -\frac{ik_0}{\mu}\left(\varepsilon\mu - \beta^2\right) E_y,\ \frac{dE_y}{dx} = -ik_0 \mu H_z.$$

Each of the systems can be transformed to a second order ODE for the leading component and two other equations for straightforward computation of the complementary electromagnetic field components. In doing so, the boundary conditions for Maxwell's equations are reduced to two pairs of boundary conditions for obtained equations. In addition, the asymptotic conditions hold for each class of waveguide modes. Thus, the problem of description of a complete set of modes in a regular planar waveguide is formulated in terms of the eigenvalues problem for the essentially self-adjoint second order differential operator:

$$-\frac{d^2\psi}{dx^2} + V\left(x\right)\psi = k^2\psi.$$

For the operator, we find some results about its spectrum, complete sets of solutions, and diagonalization by an isometric isomorphism (generalized Fourier transformation); new basis functions are related to initial ones by simple transformation formulas. The eigenvalues problem is equivalently reduced to the two problems (left and right) of the one-dimensional potential scattering theory by projection on the two branches of the continuous spectrum.

Keywords: waveguide propagation of electromagnetic radiation, equations of waveguide modes of regular waveguide, guided modes, radiation modes, a complete set of modes of a planar waveguide.
MSC numbers: 65Fxx, 65Hxx, 65L10, 65L15, 78A40, 78Mxx

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