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Retrial inventory system with multiple working vacations and two types of customers

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Abstract. In this article, we consider a continuous review perishable inventory system with retrial demands. The maximum storage capacity is S. We assume that the replenishment of inventory is instantaneous. The life time of each item is assumed to be exponential. We assume that not all customers would be requiring service on items. Hence we propose to have two types of customers, say, high priority and low priority. The high priority customer demands a unit item with require service on the demanded item before accepting it and the low priority customer demands a unit item but do not require any service on his demanded item (i.e., service time is zero). The customers arrive based on two homogeneous independent Poisson processes. Retrial is introduced for high priority customers only. The arriving high priority customer who finds the server is busy joins an orbit of unsatisfied customers. The orbiting customers compete for service by sending out signals that are exponentially distributed. The single server takes a working vacation at times when customers being served depart from the system and no customers are in the orbit. The duration of server working vacation follows an exponential distribution. At the end of each working vacation, the server only takes another new vacation if there is no any new high priority customer or repeated customer from the orbit. The joint probability distribution of the number of customer in the orbit and the inventory level is obtained for the steady state case. Some important system performance measures and the long-run total expected cost rate are derived in the steady state. Several numerical examples are presented to illustrate the effect of the system parameters and costs on these measures.

Keywords: Continuous review inventory system, Perishable item, Service facility, Multiple working vacation, Retrial, Two types of customers

MSC numbers: 60J27, 90B05

1. Introduction

Research on queueing systems with inventory control has captured much attention of researchers over the last decades. In this system, customers arrive at the service facility one by one and require service. In order to complete the customer service, an item from the inventory is needed. A served customer departs immediately from the system and the on - hand inventory decreases by one at the moment of service completion. This system is called a queueing - inventory system [1]. Berman and Kim [2] analyzed a queueing - inventory system with Poisson arrivals, exponential service times and zero lead times. The authors proved that the optimal policy is never to order when the system is empty. Berman and Sapna [3] studied queueing - inventory systems with Poisson arrivals, arbitrary distribution service times and zero lead times. The optimal value of the maximum allowable inventory which minimizes the long - run expected cost rate has been obtained.

Berman and Sapna [4] discussed a finite capacity system with Poisson arrivals, exponential distributed lead times and service times. The existence of a stationary optimal service policy has been proved. Berman and Kim [5] addressed an infinite capacity queueing - inventory system with Poisson arrivals, exponential distributed lead times and service times. The authors identified a replenishment policy which maximized the system profit. Berman and Kim [6] studied internet based supply chains with Poisson arrivals, exponential service times, the Erlang lead times and found that the optimal ordering policy has a monotonic threshold structure. The M/M/1 queueing - inventory system with backordering was investigated by Schwarz and Daduna [7]. The authors derived the system steady state behavior under $\Pi(1)$ reorder policy which is (0, Q) policy with an additional threshold 1 for the queue length as a decision variable. Krishnamoorthy et al., [8] introduced an additional control policy (N-policy) into (s, S) inventory system with positive service time.

In many real world queueing inventory systems, server(s) may become unavailable for a random period of time when there are no customers in the waiting line at a service completion instant. This random period of server absence, often called a server vacation can represent the time of server's performing some secondary task. This has been extensively investigated (see Tian and Zhang [9], Takagi [10, 11], and Doshi [12, 13]. Daniel and Ramanarayanan [14] have first introduced the concept of server vacation in inventory with two servers. In [15], they have studied an inventory system in which the server takes a rest when the level of the inventory is zero. Sivakumar [16] analyzed a retrial inventory system with multiple server vacations. In that paper, the author considered an (s, S) inventory system where arrivals of customers form a Poisson process and if the server finds an empty stock at the end of a vacation, he takes another vacation immediately otherwise he is ready to serve any arriving demands. Recently, Narayanan et al. [17] considered an inventory system with random positive service time. Customers arrived to the service station according to a Markovian arrival process and service times for each customers had phase-type distribution.

Recently, a class of semi-vacation policies has been introduced by Servi and

Finn. Such a vacation is called a working vacation (WV). Servi and Finn [18] studied an M/M/1 queue with multiple working vacations, and obtained the number of customers in the system and the waiting time distribution. In the classical vacation queueing (CVQ) models, during the vacation period the server doesn't continue on the original and such policy cause the loss or dissatisfaction (retrial customers) of the customers. For the working vacation policy, the server can still work during the vacation and may accomplish other assistant work simultaneously (i.e the server serves customers at a lower rate rather than completely stopping service during the service period). Obviously the working vacation queue is a generalization of the classical vacation queue and is motivated by real queueing inventory system.

In this paper, we consider a $(0, S)$ queueing- inventory system at a service facility with multiple working vacations and repeated attempts. The joint probability distribution of the number of customers in the orbit and the inventory level is obtained in the steady state case. Various system performance measures are derived and the total expected cost rate is calculated. To the best of our knowledge, this is the first time in retrial perishable inventory literature that the concept of multiple working vacations with two types of customers and $(0, S)$ ordering policy is studied.

The rest of the paper is organized as follows. In the next section, the mathematical model and the notations used in this paper are described. Analysis of the model and the steady state solutions of the model are obtained in section 3. Some key system performance measures are derived in section 4. In the section 5, we derive the total expected cost rate in the steady state. In section 6, we provide some interesting numerical examples. The last section is meant for conclusion.

2. Mathematical Model

Consider a service facility in which perishable items are stocked and the items are delivered to the demanding customers. The demand is for single item per customer. The maximum capacity of the inventory is S. We assume that the replenishment of inventory is instantaneous (i.e: $(0, S)$ ordering policy with zero lead time). The life time of each item is exponential with rate $\gamma(> 0)$. Customers arriving at the service station belong to any one of the two types such that the high priority and the low priority customers, and their arrivals belong to independent Poisson processes with parameters λ_1 and λ_2 respectively. The high priority customers require service on their demanded unit and the low priority customers do not want any service to be performed on their demanded unit. If the server is not occupied, arriving high priority customers get service immediately. We have assumed that an item of inventory that makes it into the service process cannot perish while in service.

There are two types of retrial policy considered in the literature of queuing systems:

1. The probability of a repeated attempt depends on the number of orbiting demands (classical retrial policy).

2. The probability of a repeated attempt is independent of the number of orbiting demands (constant retrial policy).

In this article we consider the classical retrial policy. More explicitly, when there are $i \geq 1$ demands in the orbit, a signal is sent out according to an exponential distribution with parameter ν which depends on i. Any high priority customer who finds the server busy upon arrival leaves the service area and joins the orbit of finite size N with probability p and is lost forever with probability $(1-p)$. The orbiting customers compete for service by sending out signals that are exponentially distributed.

The single server takes a working vacation at times when high priority customers being served depart from the system and no customers are in the orbit. Working vacation durations are exponentially distributed with parameter β. During the working vacation periods arriving high priority customers are served with rate $\mu_w < \mu_b$ where μ_b is the service rate of regular busy period. The service times of working vacation period and regular busy period are assumed to be exponentially distributed. At the end of each working vacation, the server only takes another new vacation (multiple working vacation) if there is no any new high priority customer or repeated customer from the orbit. Any arriving low priority customer, who finds the server is busy and only one item in the inventory is considered to be lost. Any arriving high priority customer, who finds the server is busy and orbit size full is considered to be lost. Various stochastic processes involved in the system are independent of each other.

2.1 Notations:

$$
\begin{aligned}
\mathbf{0} &: \text{Zero matrix} \\
[A]_{ij} &: \text{entry at } (i,j)^{th} \text{ position of a matrix A} \\
\delta_{ij} &: \begin{cases} 1, & \text{if } j = i \\ 0, & \text{otherwise} \end{cases} \\
\bar{\delta_{ij}} &: 1 - \delta_{ij} \\
k \in V_i^j &: k = i, i+1, \dots j \\
\overset{k}{\underset{i=r}{\Omega}} c_i &: \begin{cases} c_r c_{r-1} \cdots c_k & \text{if } r \geq k \\ 1 & \text{if } r < k \end{cases} \\
\mathbf{e}^T &: (1, 1, \dots, 1)
\end{aligned}
$$

3. Analysis

The system at any time t can be completely described by three integer-valued random variables: $L(t)$ denotes the inventory level at time t, $Y(t)$ denotes the state of server (the phase of the system) at time t and $X(t)$ represents the number of customers (the level of the system) in the orbit at time t.

There are four possible states of the single server as follows:
(1) the server is on a working vacation at time t and the server is not occupied. When the server is in this state $Y(t) = 0$.
(2) the server is on a working vacation at time t and the server is busy. If the server is in this state $Y(t) = 1$.
(3) the server is not on a working vacation at time t and the server is not occupied. Whenever the server is in this state $Y(t) = 2$.
(4) the server is not on a working vacation at time t and the server is busy. If the server is in this state $Y(t) = 3$.

From the assumptions made on the input and output processes, it can be shown that the stochastic process $\{(L(t), Y(t), X(t)), t \geq 0\}$ is a continuous time Markov chain with discrete state space given by $E = E_1 \cup E_2$, where

$$\begin{aligned} E_1 &: \{(i_1, i_2, i_3) \mid i_1 = 1, 2, \ldots, S,\ i_2 = 0, 1, 3,\ i_3 = 0, 1, 2, \ldots, N\}, \\ E_2 &: \{(i_1, 2, i_3) \mid i_1 = 1, 2, \ldots, S,\ i_3 = 1, 2, \ldots, N\}. \end{aligned}$$

To determine the infinitesimal generator

$$\Theta = ((\ a((i_1, i_2, i_3), (j_1, j_2, j_3))\ )), \quad (i_1, i_2, i_3), (j_1, j_2, j_3) \in E$$

of this process we use the following arguments :

(A.) We first consider $2 \leq L(t) = i_1 \leq S$ and $1 \leq X(t) = i_3 \leq N$. For this, the following events can happen in the system:

- (1A(a)) upon the arrival of a new high priority customer,
  - (1A(a).1) if the server is free (i.e.: $Y(t) = 0$ or $Y(t) = 2$), then the server changes to the busy state (i.e.: Y(t) changes either from 0 to 1 or from 2 to 3), $L(t)$ and $X(t)$ are remains unchanged. The state of the process either from $(i_1, 0, i_3)$ to $(i_1, 1, i_3)$ or from $(i_1, 2, i_3)$ to $(i_1, 3, i_3)$ with intensity of transition either $a((i_1, 0, i_3), (i_1, 1, i_3))$ or $a((i_1, 2, i_3), (i_1, 3, i_3))$ is given by λ_1.
  - (1A(a).2) if the server is occupied (i.e.: $Y(t) = 1$ or $Y(t) = 3$), then the customer goes into the orbit with probability p and lost forever with probability $(1 - p)$, $L(t)$ remains unchanged and number of customers in the orbit, $X(t)$, increases by one. Therefore, the state of the process either from $(i_1, 1, i_3)$ to $(i_1, 1, i_3 + 1)$ or from $(i_1, 3, i_3)$ to $(i_1, 3, i_3 + 1)$ with intensity of transition either $a((i_1, 1, i_3), (i_1, 1, i_3 + 1))$ or $a((i_1, 3, i_3), (i_1, 3, i_3 + 1))$ is given by $\bar{\delta}_{i_3 N} p \lambda_1$.
- (1A(b)) upon the arrival of a low priority customer,
  - (1A(b).1) a transition from $(i_1, 0, i_3)$ to $(i_1 - 1, 0, i_3)$ or from $(i_1, 2, i_3)$ to $(i_1 - 1, 2, i_3)$ with intensity of transition λ_2, when the server is free.

  - (1A(b).2) a transition from $(i_1, 1, i_3)$ to $(i_1 - 1, 1, i_3)$ or from $(i_1, 3, i_3)$ to $(i_1 - 1, 3, i_3)$ with intensity of transition λ_2, when the server is occupied.

- (2A) the departure of a high priority customer after the finish of its service, then the inventory level decreases by one, server becomes free (i.e.: Y(t) changes either from 1 to 0 or from 3 to 2) and $X(t)$ remains unchanged. The intensity of this transition either $a((i_1, 1, i_3), (i_1 - 1, 0, i_3))$ is given by μ_w or $a((i_1, 3, i_3), (i_1 - 1, 2, i_3))$ is given by μ_b.

- (3A) the status change of the server (i.e.: the end of the vacation $Y(t) = 0$ or $Y(t) = 1$), then $Y(t)$ changes either from 0 to 2 or from 1 to 3, $L(t)$ and $X(t)$ are remains unchanged. Hence, the intensity of this transition either $a((i_1, 0, i_3), (i_1, 2, i_3))$ or $a((i_1, 1, i_3), (i_1, 3, i_3))$ is given by β.

- (4A) the successful service request of a high customer from the orbit, (i.e.: $Y(t) = 0$ or $Y(t) = 2$) then $Y(t)$ changes either from 0 to 1 or from 2 to 3, $X(t)$ changes from i_3 to $i_3 - 1$ and $L(t)$ remains unchanged. Therefore, the intensity of this transition either $a((i_1, 0, i_3), (i_1, 1, i_3 - 1))$ or $a((i_1, 2, i_3), (i_1, 3, i_3 - 1))$ is given by $i_3\nu$.

- (5A) if the server is free (i.e.: $Y(t) = 0$ or $Y(t) = 2$), then any one of the i_1 items perishes. Therefore, the intensity of this transition either $a((i_1, 0, i_3), (i_1 - 1, 0, i_3))$ or $a((i_1, 2, i_3), (i_1 - 1, 2, i_3))$ is given by $i_1\gamma$.

- (6A) if the server is occupied (i.e.: $Y(t) = 1$ or $Y(t) = 3$), then any one of the $(i_1 - 1)$ items perishes (Note: An item of inventory that makes it into the service process cannot perish while in service). Hence, the intensity of this transition either $a((i_1, 1, i_3), (i_1 - 1, 1, i_3))$ or $a((i_1, 3, i_3), (i_1 - 1, 3, i_3))$ is given by $(i_1 - 1)\gamma$.

(B.) If no customer is in the orbit ($X(t) = i_3 = 0$) and $L(t) = 2 \leq i_1 \leq S$, the following events are possible in the system:

- (1B(a)) upon the arrival of a new high priority customer,
  - (1B(a).1) if the server is free (i.e.: $Y(t) = 0$), then the server changes to the busy state (i.e.: Y(t) changes either from 0 to 1), $L(t)$ and $X(t)$ are remains unchanged. The state of the process from $(i_1, 0, 0)$ to $(i_1, 1, 0)$ with intensity of transition $a((i_1, 0, 0), (i_1, 1, 0))$ is given by λ_1.
  - (1B(a).2) if the server is occupied (i.e.: $Y(t) = 1$ or $Y(t) = 3$), then the arriving customer goes into the orbit with probability p, $L(t)$ remains unchanged and number of customers in the orbit, $X(t)$, increases by one. Therefore, the intensity of transition either $a((i_1, 1, 0), (i_1, 1, 1))$ or $a((i_1, 3, 0), (i_1, 3, 1))$ is given by $p\lambda_1$.

- (1B(b)) upon the arrival of a low priority customer,

- (1B(b).1) a transition from $(i_1, 0, i_3)$ to $(i_1 - 1, 0, i_3)$ with intensity of transition λ_2, when the server is free.

- (1B(b).2) a transition from $(i_1, 1, i_3)$ to $(i_1 - 1, 1, i_3)$ or from $(i_1, 3, i_3)$ to $(i_1 - 1, 3, i_3)$ with intensity of transition λ_2, when the server is occupied.

- (2B) the departure of a high priority customer after the finish of its service, then the inventory level decreases by one, server becomes free (i.e.: Y(t) changes either from 1 to 0 or from 3 to 0) and $X(t)$ remains unchanged. The intensity of this transition either $a((i_1, 1, 0), (i_1 - 1, 0, 0))$ is given by μ_w or $a((i_1, 3, 0), (i_1 - 1, 0, 0))$ is given by μ_b.

- (3B) the status change of the server (i.e.: the end of the vacation $Y(t) = 1$), then $Y(t)$ changes from 1 to 3. Hence, the intensity of this transition $a((i_1, 1, 0), (i_1, 3, 0))$ is given by β.

- (4B) if the server is free (i.e.: $Y(t) = 0$), then any one of the $L(t) = i_1$ items perishes, $Y(t)$ and $X(t)$ are remains unchanged. Therefore, the intensity of this transition $a((i_1, 0, 0), (i_1 - 1, 0, 0))$ is given by $i_1\gamma$.

- (5B) if the server is occupied (i.e.: $Y(t) = 1$ or $Y(t) = 3$), then $L(t)$ changes from i_1 to $i_1 - 1$, $Y(t)$ and $X(t)$ are remains unchanged. Thus, the intensity of this transition either $a((i_1, 1, 0), (i_1 - 1, 1, 0))$ or $a((i_1, 3, 0), (i_1 - 1, 3, 0))$ is given by $(i_1 - 1)\gamma$.

The transition rates for any other transitions not considered above, when the inventory level is $2 \leq L(t) = i_1 \leq S$, are zero. The intensity of passage in the state (i_1, i_2, i_3), is given by

$$a((i_1, i_2, i_3), (j_1, j_2, j_3)) = - \sum_{i_1} \sum_{i_2} \sum_{\substack{i_3 \\ (i_1,i_2,i_3) \neq (j_1,j_2,j_3)}} a\left((i_1, i_2, i_3); (j_1, j_2, j_3)\right).$$

Using the above arguments, we have constructed the following matrices:

For $i_1 = 2, 3, \ldots, S$,

$$[H_{i_1}]_{i_3 j_3} = \begin{cases} i_1\gamma & j_3 = i_3, \quad i_3 \in V_0^N \\ \lambda_2 & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[J_{i_1}]_{i_3 j_3} = \begin{cases} (i_1 - 1)\gamma & j_3 = i_3, \quad i_3 \in V_0^N \\ \lambda_2 & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[K_{i_1}]_{i_3 j_3} = \begin{cases} i_1\gamma & j_3 = i_3, \quad i_3 \in V_1^N \\ \lambda_2 & j_3 = i_3, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[C_1]_{i_3 j_3} = \begin{cases} \mu_w & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[C_2]_{i_3 j_3} = \begin{cases} \mu_b & j_3 = i_3, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[C_3]_{i_3 j_3} = \begin{cases} \mu_b & j_3 = i_3, \quad i_3 = 0 \\ 0 & \text{otherwise.} \end{cases}$$

Combining theses matrices in suitable form, we get

$$[B_{i_1}]_{i_2 j_2} = \begin{cases} H_{i_1}, & j_2 = 0, \quad i_2 = 0 \\ J_{i_1}, & j_2 = i_2, \quad i_2 = 1, 3 \\ K_{i_1}, & j_2 = i_2, \quad i_2 = 2 \\ C_1, & j_2 = 0, \quad i_2 = 1 \\ C_2, & j_2 = 2, \quad i_2 = 3 \\ C_3, & j_2 = 0, \quad i_2 = 3 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

For $i_1 = 2, 3, \ldots, S$,

$$[F]_{i_3 j_3} = \begin{cases} \beta & j_3 = i_3, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[F_1]_{i_3 j_3} = \begin{cases} \beta & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[D]_{i_3 j_3} = \begin{cases} \lambda_1 & j_3 = i_3, \quad i_3 \in V_0^N \\ \nu_{i_3} & j_3 = i_3 - 1, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[D_1]_{i_3j_3} = \begin{cases} \lambda_1 & j_3 = i_3, \quad i_3 \in V_1^N \\ \nu_{i_3} & j_3 = i_3 - 1, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[L_{i_1}]_{i_3j_3} = \begin{cases} -(\nu_{i_3} + \beta\bar{\delta}_{i_30} + i_1\gamma + \lambda_1 + \lambda_2) & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[V_{i_1}]_{i_3j_3} = \begin{cases} -(\mu_w + \beta + p\lambda_1 + (i_1 - 1)\gamma + \lambda_2) & j_3 = i_3, \quad i_3 \in V_0^N \\ p\lambda_1 & j_3 = i_3 + 1, \quad i_3 \in V_0^{N-1} \\ 0 & \text{otherwise.} \end{cases}$$

$$[R_{i_1}]_{i_3j_3} = \begin{cases} -(\nu_{i_3} + i_1\gamma + \lambda_1 + \lambda_2) & j_3 = i_3, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[T_{i_1}]_{i_3j_3} = \begin{cases} -(\mu_b + p\lambda_1 + (i_1 - 1)\gamma + \lambda_2) & j_3 = i_3, \quad i_3 \in V_0^N \\ p\lambda_1 & j_3 = i_3 + 1, \quad i_3 \in V_0^{N-1} \\ 0 & \text{otherwise.} \end{cases}$$

Hence the matrix $A_{i_1}(i_1 = 2, 3, \ldots, S)$ is given by

$$[A_{i_1}]_{i_2j_2} = \begin{cases} F, & j_2 = 2, \quad i_2 = 0 \\ F_1, & j_2 = 3, \quad i_2 = 1 \\ D, & j_2 = 1, \quad i_2 = 0 \\ D_1, & j_2 = 3, \quad i_2 = 2 \\ L_{i_1}, & j_2 = 0, \quad i_2 = 0 \\ V_{i_1}, & j_2 = i_2, \quad i_2 = 1 \\ R_{i_1}, & j_2 = i_2, \quad i_2 = 2 \\ T_{i_1}, & j_2 = i_2, \quad i_2 = 3 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

(C.) Now we consider $L(t) = i_1 = 1$ and $1 \leq X(t) = i_3 \leq N$. For this, the following events can happen in the system:

- (1C(a)) upon the arrival of a new customer,
  - (1C(a).1) if the server is free (i.e.: $Y(t) = 0$ or $Y(t) = 2$), then the server changes to the busy state (i.e.: Y(t) changes either from 0 to 1 or from 2 to 3). Arguments similar to above type (1A(a).1).
  - (1C(a).2) if the server is occupied (i.e.: $Y(t) = 1$ or $Y(t) = 3$), then the customer goes into the orbit. Thus, the arguments similar to above type (1A(a).2).
- (1C(a)) upon the arrival of a low priority customer,
  - the arguments similar to above type (1A(b)).

- (2C) the departure of a request after the finish of its service, then the server becomes free (i.e.: Y(t) changes either from 1 to 0 or from 3 to 2), inventory level goes to level S (instantaneous reorder) and $X(t)$ remains unchanged. This is the transition of type (2A).
- (3C) the status change of the server (i.e.: the end of the vacation), then $Y(t)$ changes either from 0 to 2 or from 1 to 3 since $L(t)$ and $X(t)$ are remains unchanged. It is the transition of type (3A).
- (4C) the successful service request of a customer from the orbit, then $Y(t)$ changes either from 0 to 1 or from 2 to 3, customer level in the orbit decreases by one and $L(t)$ remains unchanged. Therefore, the transition of type (4A).
- (5C) if the server is free, then the item perish and inventory level goes to from 1 to S, $Y(t)$ and $X(t)$ are remains unchanged. It is the transition of type (5A).

(D.) If no customer is in the orbit (i.e.: $X(t) = i_3 = 0$) and $L(t) = 1$, then in this case all arguments are similar to above types ((1B(a)), (1B(b)), (2B), (3B), (4B)).

The transition rates for any other transitions not considered above ((C),(D)), when the inventory level is one, are zero. The intensity of passage in the state $(1, i_2, i_3)$, is given by

$$-\sum_{(1,i_2,i_3)\neq(1,j_2,j_3)} a\left((1, i_2, i_3), (1, j_2, j_3)\right).$$

Using the above arguments, we have constructed the following matrices:

$$[C_0]_{i_3 j_3} = \begin{cases} \gamma & j_3 = i_3, \quad i_3 \in V_0^N \\ \lambda_2 & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[C]_{i_3 j_3} = \begin{cases} \gamma & j_3 = i_3, \quad i_3 \in V_1^N \\ \lambda_2 & j_3 = i_3, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

Hence the matrix B_1 is given by

$$[B_1]_{i_2 j_2} = \begin{cases} C_0, & j_2 = 0, \quad i_2 = 0 \\ C, & j_2 = 2, \quad i_2 = 2 \\ C_1, & j_2 = 0, \quad i_2 = 1 \\ C_2, & j_2 = 2, \quad i_2 = 3 \\ C_3, & j_2 = 0, \quad i_2 = 3 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

$$[G_1]_{i_3j_3} = \begin{cases} -(\nu_{i_3} + \beta\bar{\delta}_{i_30} + \gamma + \lambda_1 + \lambda_2) & j_3 = i_3, \quad i_3 \in V_0^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[G_2]_{i_3j_3} = \begin{cases} -(\mu_w + \beta + p\lambda_1) & j_3 = i_3, \quad i_3 \in V_0^N \\ p\lambda_1 & j_3 = i_3 + 1, \quad i_3 \in V_0^{N-1} \\ 0 & \text{otherwise.} \end{cases}$$

$$[G_3]_{i_3j_3} = \begin{cases} -(\nu_{i_3} + \gamma + \lambda_1 + \lambda_2) & j_3 = i_3, \quad i_3 \in V_1^N \\ 0 & \text{otherwise.} \end{cases}$$

$$[G_4]_{i_3j_3} = \begin{cases} -(p\lambda_1 + \mu_b) & j_3 = i_3, \quad i_3 \in V_0^N \\ p\lambda_1 & j_3 = i_3 + 1, \quad i_3 \in V_0^{N-1} \\ 0 & \text{otherwise.} \end{cases}$$

Combining theses matrices in suitable form, we get

$$[A_1]_{i_2j_2} = \begin{cases} D, & j_2 = 1, \quad i_2 = 0 \\ F, & j_2 = 2, \quad i_2 = 0 \\ F_1, & j_2 = 3, \quad i_2 = 1 \\ D_1, & j_2 = 3, \quad i_2 = 2 \\ G_1, & j_2 = 0, \quad i_2 = 0 \\ G_2, & j_2 = 1, \quad i_2 = 1 \\ G_3, & j_2 = 2, \quad i_2 = 2 \\ G_4, & j_2 = 3, \quad i_2 = 3 \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

Therefore, the matrix Θ can be written in the following form:

$$[\Theta]_{i_1j_1} = \begin{cases} A_{i_1} & j_1 = i_1, \ i_1 = 1, 2, \ldots, S \\ B_{i_1} & j_1 = i_1 - 1, \ i_1 = 2, \ldots, S-1, S \\ B_1 & j_1 = S, \ i_1 = 1, \\ \mathbf{0} & \text{Otherwise.} \end{cases}$$

More explicitly,

$$[\Theta]_{i_1j_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ S-1 \\ S \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & \cdots & S-1 & S \end{array} \\ \begin{pmatrix} A_1 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & B_1 \\ B_2 & A_2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & B_3 & A_3 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & A_{S-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & B_S & A_S \end{pmatrix} \end{array}$$

It can be noted that the matrices A_{i_1} and B_{i_1}, $i_1 = 1, 2, \ldots, S$, are square matrices of size $(4N + 3)$. R_{i_1}, K_{i_1}, G_3, C and D_1, $i_1 = 1, 2, 3, \ldots, S$, are square matrices of size N. F and C_2 are matrices of size $(N + 1) \times N$. D_1 is a matrix of size $N \times (N + 1)$. All other matrices are square matrices of order $N + 1$.

3.1 Steady State Analysis

It can be seen from the structure of Θ that the homogeneous Markov process $\{(L(t), Y(t), X(t)), t \geq 0\}$ on the finite space E is irreducible. Hence the limiting distribution,

$$\phi^{(i_1,i_2,i_3)} = \lim_{t\to\infty} Pr[L(t) = i_1, Y(t) = i_2, X(t) = i_3 | L(0), Y(0), X(0)], \text{exists}.$$

Let $\mathbf{\Phi} = (\mathbf{\Phi^{(1)}}, \ldots, \mathbf{\Phi^{(S)}})$, each vector $\mathbf{\Phi^{(i_1)}}$ being partitioned as follows

$$\mathbf{\Phi^{(i_1)}} = (\mathbf{\Phi^{(i_1,0)}}, \mathbf{\Phi^{(i_1,1)}}, \mathbf{\Phi^{(i_1,2)}}, \mathbf{\Phi^{(i_1,3)}}), \qquad i_1 = 1, 2, \ldots, S;$$

where

$$\begin{aligned}
\mathbf{\Phi^{(i_1,i_2)}} &= (\phi^{(i_1,i_2,0)}, \phi^{(i_1,i_2,1)}, \ldots, \phi^{(i_1,i_2,N)}), \qquad i_1 = 1, 2, \ldots, S;\ i_2 = 0, 1, 3; \\
\mathbf{\Phi^{(i_1,2)}} &= (\phi^{(i_1,2,1)}, \phi^{(i_1,2,2)}, \ldots, \phi^{(i_1,2,N)}), \qquad i_1 = 1, 2, \ldots, S.
\end{aligned}$$

Then the vector of limiting probabilities $\mathbf{\Phi}$ satisfies

$$\mathbf{\Phi}\Theta = \mathbf{0} \quad \text{and} \quad \sum\sum_{(i_1,i_2,i_3)}\sum \phi^{(i_1,i_2,i_3)} = 1. \qquad (*)$$

Theorem : The limiting distribution $\mathbf{\Phi}$ is given by

$$\mathbf{\Phi^{(i_1)}} = \mathbf{\Phi^{(1)}}\mathbf{\Pi_{i_1}}, \quad i_1 = 1, \ldots, S,$$

where

$$\mathbf{\Pi_{i_1}} = (-1)^{i_1-1} \underset{k=1}{\overset{r=i_1-1}{\Omega}} A_k B_{k+1}^{-1}, \quad i_1 = 1, 2, \ldots, S.$$

The value of $\mathbf{\Phi^{(1)}}$ can be obtained from the relation

$$\mathbf{\Phi^{(1)}}\left[B_1 + \left\{(-1)^{S-1} \underset{r=1}{\overset{S-1}{\Omega}} A_r B_{r+1}^{-1}\right\}\right] = \mathbf{0},$$

and

$$\mathbf{\Phi^{(1)}}\left[\sum_{i_1=2}^{S}\left((-1)^{i_1-1} \underset{r=1}{\overset{i_1-1}{\Omega}} A_r B_{r+1}^{-1}\right) + I\right]\mathbf{e} = 1.$$

Proof:

The first equation of $(*)$ yields the following set of equations :

$$\begin{aligned}
\mathbf{\Phi}^{(i_1)} A_{i_1} + \mathbf{\Phi}^{(i_1+1)} B_{i_1+1} &= \mathbf{0}, \qquad i_1 = 1, 2, \ldots, S-1, \\
\mathbf{\Phi}^{(S)} A_S + \mathbf{\Phi}^{(1)} B_1 &= \mathbf{0}.
\end{aligned}$$

Solving the above set of equations we get the required solution. □

4. Performance Measures of the System

In this section some performance measures of the system under consideration in the steady state are derived.

4.1 Expected Inventory Level

Let η_{II} denote the expected inventory level in the steady state. Since $\Phi^{(i_1)}$ is the steady state probability vector that there are i_1 items in the inventory with each component represents a particular combination of the number of customers in the orbit and the status of server, $\Phi^{(i_1)}\mathbf{e}$ gives the probability of i_1 item in the inventory in the steady state. Hence η_{II} is given by

$$\eta_{II} = \sum_{i_1=1}^{S} i_1 \Phi^{(i_1)}\mathbf{e}.$$

4.2 Expected Reorder Rate

Let η_{RR} denote the expected reorder rate in the steady state. We note that a reorder is triggered when the inventory level drops from 1 to 0. This will occur when

1. a failure of one item or arrival of low priority customer when the server is on a working vacation and the server is not occupied or
2. a failure of one item or arrival of low priority customer when the server is not on a working vacation and the server is not occupied or
3. a service completion of the customer when the server is on a working vacation and the server is busy or
4. a service completion of the customer when the server is not on a working vacation and the server is busy

This leads to

$$\eta_{RR} = \sum_{i_3=0}^{N}(\lambda_2+\gamma)\phi^{(1,0,i_3)} + \sum_{i_3=1}^{N}(\lambda_2+\gamma)\phi^{(1,2,i_3)} + \sum_{i_3=0}^{N}\mu_w\phi^{(1,1,i_3)} + \sum_{i_3=0}^{N}\mu_b\phi^{(1,3,i_3)}.$$

4.3 Expected Perishable Rate

Since $\phi^{(i_1,i_2,i_3)}$ is a vector of probabilities with the inventory level is i_1, status of the server is i_2 and the number of customer in the orbit is i_3, the expected perishable rate η_{FR} in the steady state is given by

$$\eta_{FR} = \sum_{i_1=1}^{S}\sum_{i_3=0}^{N} i_1\gamma\phi^{(i_1,0,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_1\gamma\phi^{(i_1,2,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=0}^{N}(i_1-1)\gamma(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}).$$

4.4 Expected Number of High Priority Customers in the Orbit

Let η_{OR} denote the expected number of customers in the orbit in the steady state. Thus we get

$$\eta_{OR} = \sum_{i_1=1}^{S}\sum_{i_2=0}^{3}\sum_{i_3=1}^{N} i_3 \phi^{(i_1,i_2,i_3)}.$$

4.5 Expected Number of High Priority Customers Lost When $0 \leq X(t) \leq N-1$

Let η_{HL1} denote the expected number of high priority customers lost before entering the orbit in the steady state. Thus we get

$$\eta_{HL1} = \sum_{i_1=1}^{S}\sum_{i_3=0}^{N-1}(1-p)\lambda_1\phi^{(i_1,1,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=0}^{N-1}(1-p)\lambda_1\phi^{(i_1,3,i_3)}.$$

4.6 Expected Number of High Priority Customers Lost When $X(t) = N$

Let η_{HL2} denote the expected blocking rate for high priority customers in the steady state. Thus we get

$$\eta_{HL2} = \sum_{i_1=1}^{S}\lambda_1\phi^{(i_1,1,N)} + \sum_{i_1=1}^{S}\lambda_1\phi^{(i_1,3,N)}.$$

4.7 Expected Number of Low Priority Customers Lost

Let η_{LL} denote the expected number of low priority customers lost in the steady state. Thus we get

$$\eta_{LL} = \sum_{i_3=0}^{N}\lambda_2\phi^{(1,1,i_3)} + \sum_{i_3=0}^{N}\lambda_2\phi^{(1,3,i_3)}.$$

4.8 Probability that Server is Idle

Let η_{SI} denote the probability that server is idle is given by

$$\eta_{PI} = \sum_{i_1=1}^{S}\sum_{i_3=0}^{N}\phi^{(i_1,0,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N}\phi^{(i_1,2,i_3)}.$$

4.9 Probability that Server is Busy

Let η_{PB} denote the probability that server is busy is given by

$$\eta_{PB} = \sum_{i_1=1}^{S}\sum_{i_3=0}^{N}(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}).$$

4.10 The Probability that Server is Working in Vacation Period

Let η_{FV} denote the probability that the server is working in vacation period in the steady state and is given by

$$\eta_{FV} = \sum_{i_1=1}^{S}\sum_{i_3=0}^{N} \phi^{(i_1,1,i_3)}.$$

4.11 The Probability that Server is Working in Normal Period

Let η_{FV} denote the probability that the server is working in normal period in the steady state and is given by

$$\eta_{NV} = \sum_{i_1=1}^{S}\sum_{i_3=0}^{N} \phi^{(i_1,3,i_3)}.$$

4.12 The Overall Rate of Retrials

Let η_{OR} denote the overall rate of retrials in the steady state. Then

$$\eta_{OR} = \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3\nu\phi^{(i_1,0,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3\nu\phi^{(i_1,2,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3\nu(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}).$$

4.13 The Successful Retrial Rate

Let η_{SR} denote the successful retrial rate in the steady state. Then

$$\eta_{SR} = \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3\nu\phi^{(i_1,0,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3\nu\phi^{(i_1,2,i_3)}.$$

4.14 The Fraction of Successful Rate of Retrial

Let η_{FR} denote the fraction of successful retrial rate in the steady state. Then

$$\eta_{FR} = \frac{\eta_{SR}}{\eta_{OR}}.$$

5. Optimal Cost Analysis

In this section, we discuss the problem of minimizing the steady-state expected cost rate under the following cost structure.

c_h : The inventory carrying cost per unit item per unit time
c_s : Set up cost (ordering cost) per order
c_p : Failure cost per unit item per unit time
c_w : Waiting time cost of a high priority customer per unit time
c_r : Cost due to loss of high priority customers per unit per unit time
c_l : Cost due to loss of low priority customers per unit per unit time

Then the long run total expected cost rate is given by

$$TC(S,N) = c_h\eta_{II} + c_s\eta_{RR} + c_p\eta_{FR} + c_w\eta_{OR} + c_r(\eta_{HL} = \eta_{HL1} + \eta_{HL2}) + c_l\eta_{LL}.$$

Substituting the values of η's we get TC(S, N)=

$$c_h \sum_{i_1=1}^{S} i_1\phi^{(i_1)}\mathbf{e} + c_s\left[\sum_{i_3=0}^{N}(\lambda_2+\gamma)\phi^{(1,0,i_3)} + \sum_{i_3=1}^{N}(\lambda_2+\gamma)\phi^{(1,2,i_3)}\right] +$$
$$c_s\left[\sum_{i_3=0}^{N}\mu_w\phi^{(1,1,i_3)} + \sum_{i_3=0}^{N}\mu_b\phi^{(1,3,i_3)}\right] + c_p\left[\sum_{i_1=1}^{S}\sum_{i_3=0}^{N} i_1\gamma\phi^{(i_1,0,i_3)}\right] +$$
$$c_p\left[\sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_1\gamma\phi^{(i_1,2,i_3)} + \sum_{i_1=1}^{S}\sum_{i_3=0}^{N}(i_1-1)\gamma(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)})\right] +$$
$$c_w\left[\sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3(\phi^{(i_1,0,i_3)} + \phi^{(i_1,2,i_3)}) + \sum_{i_1=1}^{S}\sum_{i_3=1}^{N} i_3(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)})\right] +$$
$$c_l\sum_{i_3=0}^{N}\lambda_2(\phi^{(1,1,i_3)} + \phi^{(1,3,i_3)}) + c_r\sum_{i_1=1}^{S}\lambda_1(\phi^{(i_1,1,N)} + \phi^{(i_1,3,N)}) +$$
$$c_r\sum_{i_1=1}^{S}\sum_{i_3=0}^{N-1}(1-p)\lambda_1(\phi^{(i_1,1,i_3)} + \phi^{(i_1,3,i_3)}).$$

Due to the complex form of the limiting distribution, it is difficult to discuss the qualitative behavior of the cost function $TC(S,N)$ analytically. Hence, a detailed computational study of the expected cost rate function is carried out in the next section.

6. Numerical Illustrations

In this section, we perform some numerical experiments to study the effect of various parameters on the system performance measures such as the expected number of high priority customers in the orbit and expected number of low priority customers loss.

Example 1.

In this example, we study the impact of the setup cost c_s, holding cost c_h, perishable cost c_p, waiting time cost of high priority customer c_w, shortage cost of high priority customer c_r and shortage cost of low priority customer c_l on the total expected cost rate TC. Towards this end, we first fix the parameter values as $\lambda_1 = 0.01,\ \lambda_2 = 0.03, \beta = 0.03,\ \gamma = 2,\ \nu = 0.003, \mu_w = 11,\ \mu_b = 20.$ We observe the following from Tables 1 to 2:

1. The total expected cost rate TC increases, when c_s, c_h, c_p, c_w, c_r and c_l increase. TC is more sensitive to c_p than to c_s, c_h, c_w, c_r and c_l.

2. The total expected cost rate TC increases, when N increases.

3. The total expected cost rate TC decreases, when S increases.

Table 1: Effect of c_h, c_s, c_p, c_w on the total expected cost

c_h	TC	c_s	TC	c_p	TC	c_w	TC
0.001	146.353059	50	146.353059	1	146.353059	0.3	146.353059
0.002	146.356507	55	158.574681	3	194.619804	0.5	146.353062
0.003	146.359956	60	170.796303	5	242.886549	0.7	146.353065
0.004	146.363404	65	183.017925	7	291.153294	0.9	146.353068
0.005	146.366853	70	195.239547	9	339.420039	1.1	146.353071
0.006	146.370302	75	207.461169	11	387.686784	1.3	146.353074

Table 2: Effect of c_r, c_l, N, S on the total expected cost

c_r	TC	c_l	TC	N	TC	S	TC
0.1	146.353059	0.02	146.353059	4	0.000051	10	0.000051
0.3	146.353060	0.04	146.353071	5	0.000061	11	0.000050
0.5	146.353061	0.06	146.353084	6	0.000072	12	0.000048
0.7	146.353062	0.08	146.353109	7	0.000082	13	0.000047
0.9	146.353063	0.10	146.353122	8	0.000092	14	0.000046
1.1	146.353064	0.12	146.353135	9	0.000103	15	0.000045

Example 2.

In this example, we look at the impact of the parameters λ_1, λ_2, μ_w, μ_b, γ, ν and β, on the total expected cost rate. For this, we first fix the cost values as $c_h = 0.001$, $c_s = 50$, $c_p = 1$, $c_w = 0.3$, $c_r = 0.1$ and $c_l = 0.02$. The assumed values for the other parameters are shown in the Figures and Figure captions.

1. The effect of different values of λ_1, λ_2 and γ on the TC is shown in Figures 1 − 3. Figures 1 − 3 reveal that TC increases as λ_1, λ_2 and γ increase.

2. The effect of different values of β, ν and λ_1 on the TC is depicted in Figures 4 and 5. From Figures 4 and 5, we observe that TC decreases as β and ν increase.

3. The effect of different values of μ_w, μ_b and ν on the TC is shown in Figures 6 and 7. From Figures 6 and 7, we observe that μ_w, μ_b and ν increase as TC decreases.

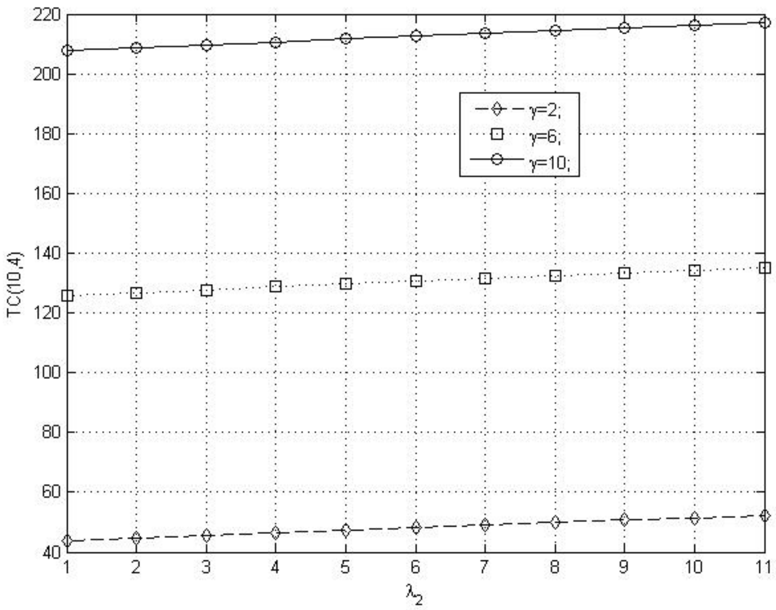

Figure 1: TC vs λ_2 for different values of γ
$(\lambda_1 = 0.01,\ \beta = 0.3,\ \nu = 0.4,\ \mu_w = 11,\ \mu_b = 20,\ p = 0.5)$

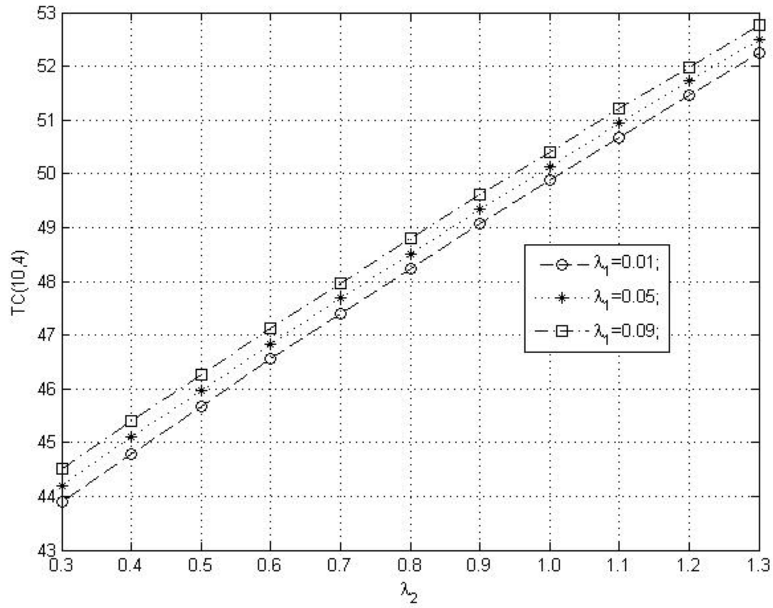

Figure 2: TC vs λ_2 for different values of λ_1
$(\beta = 0.3,\ \gamma = 2,\ \nu = 0.4,\ \mu_w = 11,\ \mu_b = 20,\ p = 0.5)$

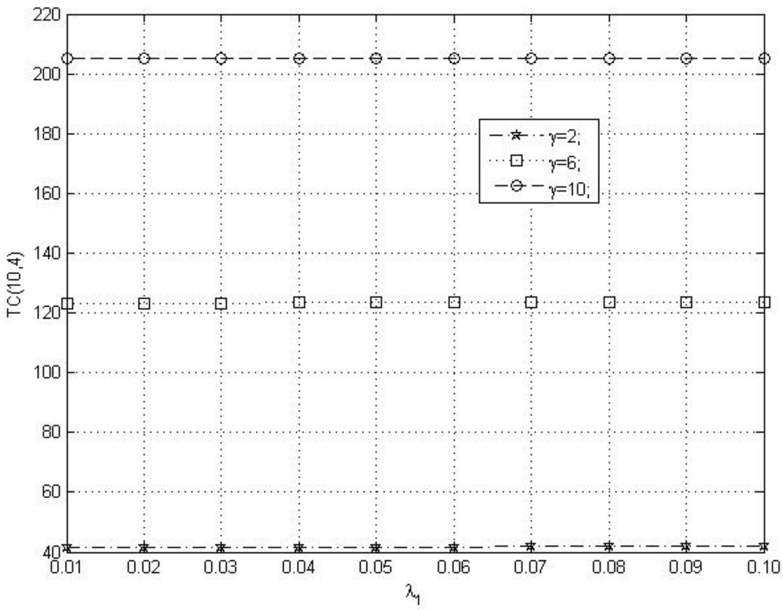

Figure 3: TC vs λ_1 for different values of γ
($\lambda_2 = 0.03,\ \beta = 0.3,\ \nu = 0.4,\ \mu_w = 11,\ \mu_b = 20, p = 0.5$)

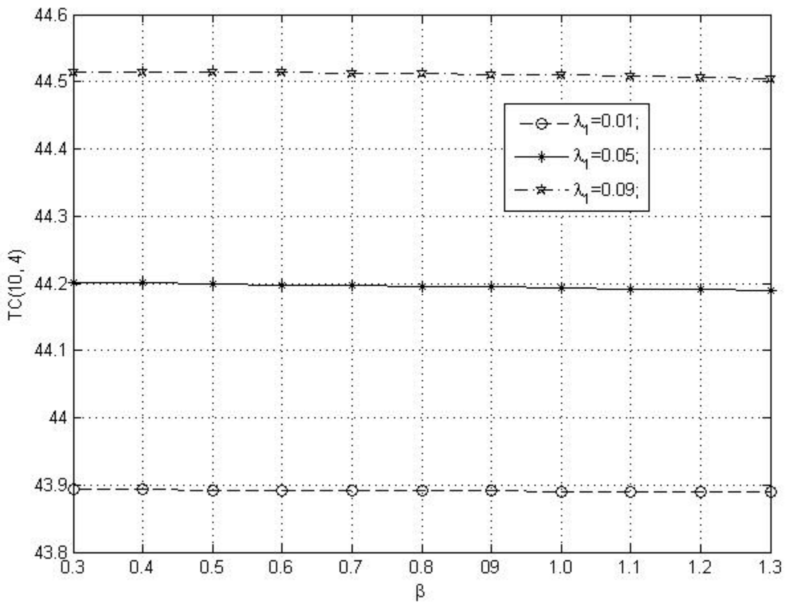

Figure 4: TC vs β for different values of λ_1
($\lambda_2 = 0.3,\ \gamma = 2,\ \nu = 0.4,\ \mu_w = 11,\ \mu_b = 20, p = 0.5$)

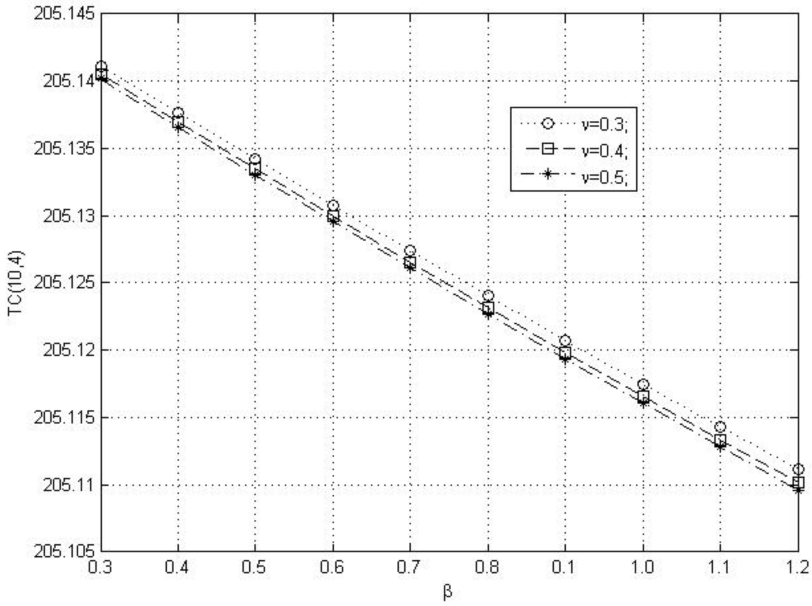

Figure 5: TC vs β for different values of ν
$(\lambda_1 = 0.01,\ \lambda_2 = 0.03,\ \gamma = 2,\ \nu = 0.5,\ \mu_w = 11,\ \mu_b = 20, p = 0.5)$

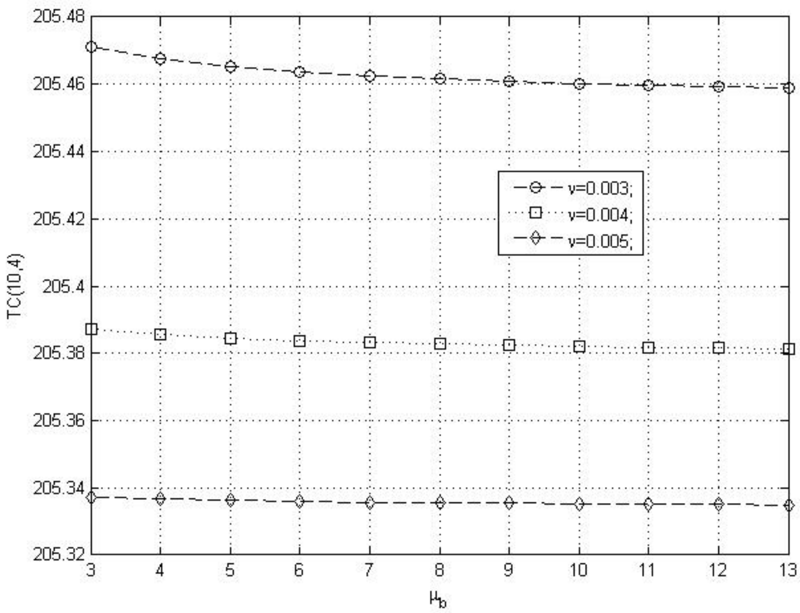

Figure 6: TC vs μ_b for different values of ν
$(\lambda_1 = 0.01,\ \lambda_2 = 0.03,\ \beta = 0.3,\ \gamma = 2,\ \mu_w = 2,\ p = 0.5)$

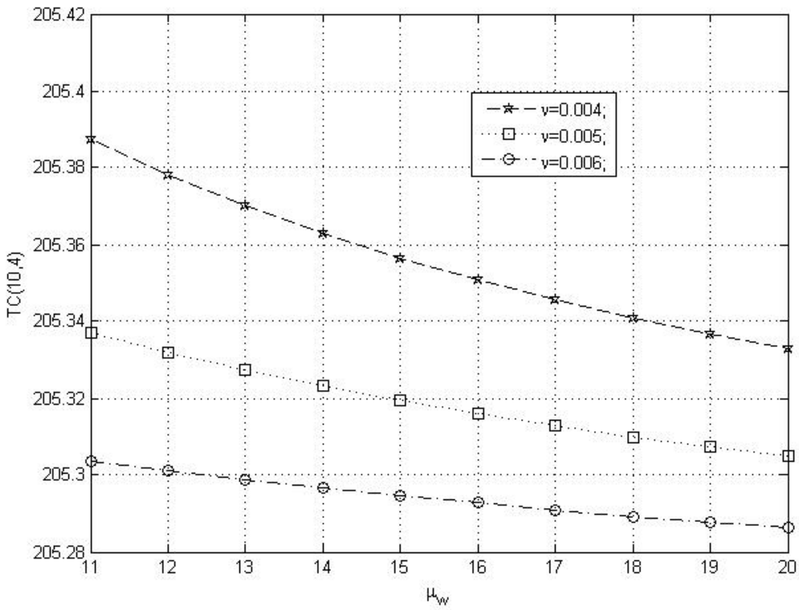

Figure 7: TC vs μ_w for different values of ν
($\lambda_1 = 0.01,\ \lambda_2 = 0.03,\ \beta = 0.3,\ \gamma = 2,\ \mu_b = 20,\ p = 0.5$)

Example 3.

Here, we study the impact of the parameters λ_1, μ_w and μ_b, on the expected number of high priority customers in the orbit, η_{OR}. Towards this end, we first fix the cost values as $c_h = 0.001$, $c_s = 50$, $c_p = 1$, $c_w = 0.3$, $c_r = 0.1$ and $c_l = 0.02$.

1. The effect of different values of λ_1, μ_w and μ_b, on the η_{OR} is shown in Figures 8 and 9. From Figures 8 and 9, we observe that
   (a) η_{OR} increases as λ_1 increases and
   (b) μ_w and μ_b increases as η_{OR} decreases.

Example 4.

In this example, we study to illustrate the effect of the parameters λ_1, μ_w and μ_b, on the expected number of high priority customers lost. Towards this end, We first select the cost values as $c_h = 0.001$, $c_s = 50$, $c_p = 1$, $c_w = 0.3$, $c_r = 0.1$ and $c_l = 0.02$. The results are presented from Figures 10 through 11.

1. η_{HL} increases as λ_1 increases and
2. μ_w and μ_b increases as η_{HL} decreases.

Finally, the effect of arrival rate of low priority customer, λ_1 and perishable rate, γ, on the expected lost rate of low priority customer is presented in Figure 12.

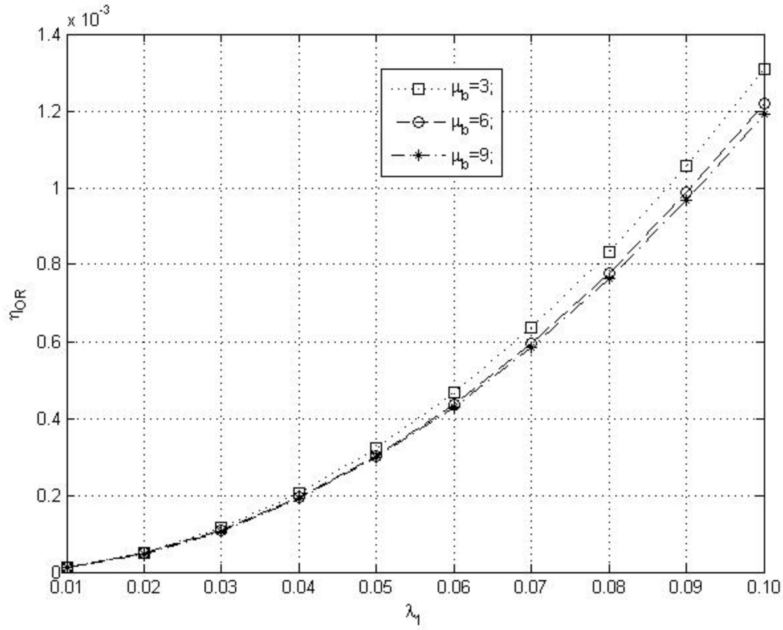

Figure 8: η_{OR} vs λ_1 for different values of μ_b
$(\lambda_2 = 0.03,\ \beta = 0.3,\ \gamma = 2,\ \nu = 0.4,\ \mu_w = 2,\ p = 0.5)$

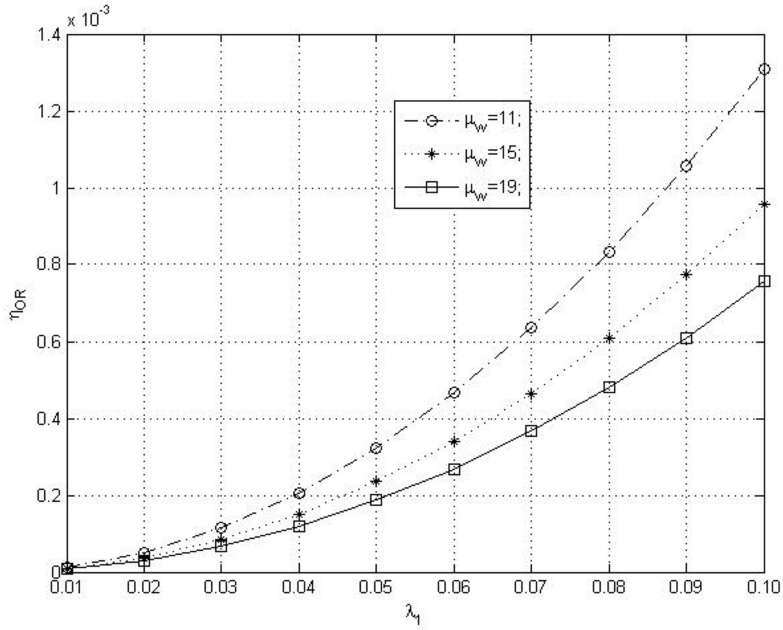

Figure 9: η_{OR} vs λ_1 for different values of μ_w
$(\lambda_2 = 0.03,\ \beta = 0.3,\ \gamma = 2,\ \nu = 0.4,\ \mu_b = 20,\ p = 0.5)$

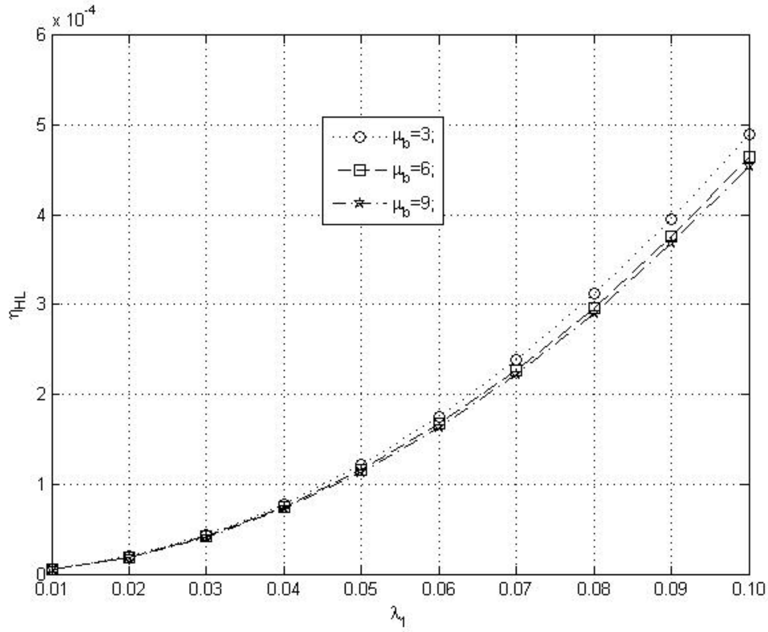

Figure 10: η_{HL} vs λ_1 for different values of μ_b
$(\lambda_2 = 0.03,\ \beta = 0.3,\ \gamma = 2,\ \nu = 0.4,\ \mu_w = 2,\ p = 0.5)$

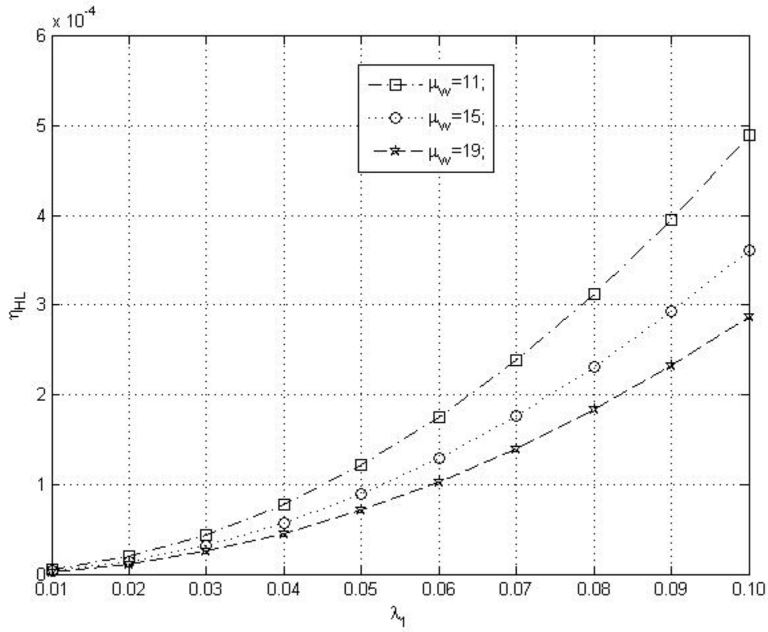

Figure 11: η_{HL} vs λ_1 for different values of μ_w
$(\lambda_2 = 0.03,\ \beta = 0.3,\ \gamma = 2,\ \nu = 0.4,\ \mu_b = 20,\ p = 0.5)$

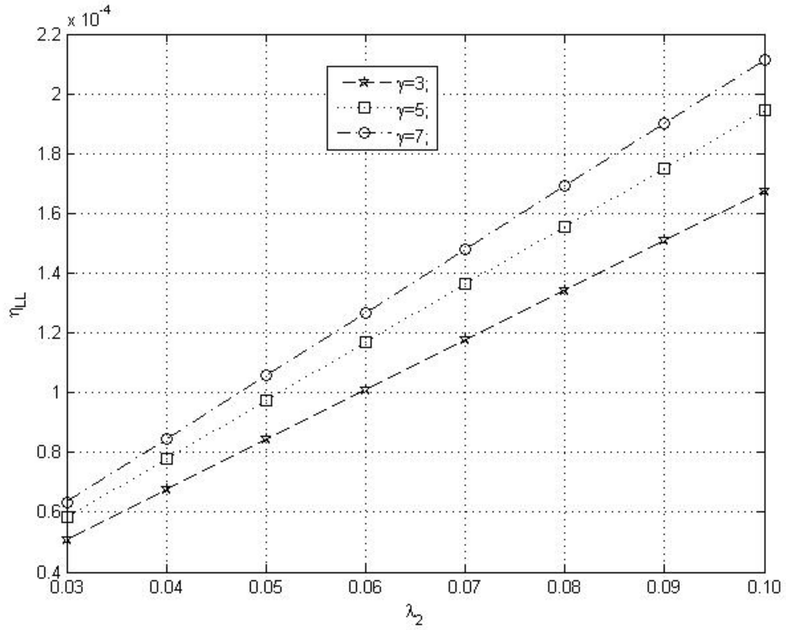

Figure 12: η_{LL} vs λ_2 for different values of γ
$(\lambda_1 = 0.01,\ \beta = 0.3,\ \nu = 0.4,\ \mu_w = 11,\ \mu_b = 20,\ p = 0.5)$

7. Conclusion

In this article, we analyzed a continuous review stochastic retrial inventory system with Poisson inputs, multiple working vacation and two types of customers. The service times and the retrial demand time points form independent exponential distributions. The model is analyzed within the framework of Markov processes. Joint probability distribution of the number of customers in the orbit and the inventory level is obtained in the steady state. Various system performance measures are derived and the long-run total expected cost rate is calculated. By assuming a suitable cost structure on the inventory system, we have presented extensive numerical illustrations to show the effect of change of values for constants on the total expected cost rate. The authors are working in the direction of MAP (Markovian arrival process) arrival for the two types of customers and service times follow PH-distributions.

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