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Integration of highly oscillatory functions

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Abstract. Ability to calculate integrals of rapidly oscillating functions is crucial for solving many problems in optics, electrodynamics, quantum mechanics, nuclear physics, and many other areas. The article considers the method of computing oscillatory integrals with the help of the transition to the numerical solution of the system of ordinary differential equations without boundary conditions. Using the differentiation matrices reduces the problem to solving a system of linear algebraic equations.

We have proposed several variants of constructing differentiation matrices, leading to effective and sustainable methods of solving the systems of linear equations with the subsequent calculation of integrals of rapidly oscillating functions for a wide class of nonlinear and even non monotonic phase functions. We have also proposed a simple option for computing integrals of rapidly oscillating functions in the case of linear phase functions that allows us to express this is almost analytically. The advantages of the proposed methods are demonstrated by a number of numerical examples.

Keywords: integration of rapidly oscillating functions, Filon method, Levin method, Chebyshev differential matrix, ill-conditioned matrices, sustainable methods for solving systems of linear algebraic equations

MSC numbers: 65D32; 41A55; 60D05

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