Modeling of $^{6}_{\Lambda}$He hypernucleus within configuration space Faddeev approach

I. Filikhin[a], V. M. Suslov and B. Vlahovic

Department of Physics, North Carolina Central University, 1801 Fayetteville Street, Durham, NC 27707, USA

e-mail: [a] ifilikhin@nccu.edu

Abstract. The cluster ^{4}He $+ \Lambda +$ n model is applied to describe the $^{6}_{\Lambda}$He hypernucleus. The consideration is based on the configuration space Faddeev equations for a system of non-identical particles. A set of the pair potentials includes the OBE simulating (NSC97f) model for the Λn interaction and the phenomenological potentials for the $\alpha\Lambda$ and αn interactions. We calculated energies of ($1^{-},2^{-}$) spin doublet. For the 2^{-} excitation energy, the obtained value is 0.18 MeV. The hyperon binding energy of the bound 1^{-} state is less than the experimental value, which may be an evidence for violation of the exact three-body cluster structure.

1. Introduction

The spin doublet 1^-, 2^- of the $^6_\Lambda$He hypernucleus is of a great interest for testing the theoretical models of the hyperon-nucleon interaction, in particular, for the study of spin-dependence of the interaction [1, 2, 3]. Especially, this study is important in relation to the recently observed the $^7_\Lambda$He [4], $^6_\Lambda$H hypernuclei [5] and possible bound Λn system [6].

The experimental value is known for hyperon binding energy of the $^6_\Lambda$He ground state [7]. The binding energy E_n of the $^6_\Lambda$He with respect to the two-particle decay accompanied by the separation of the loosely bound neutron was measured to be $E_n=0.17\pm0.10$ MeV. The parameters of the 2^- resonance state are not measured yet.

Theoretical considerations for the states have been done by Motoba et al. [2] and Hiyama et al. [3] using the three-body cluster $\alpha\Lambda n$ model. Indirect prediction for the $(1^-, 2^-)$ energy spacing was given in our work [8]. In Ref. [2], it was found that the spin-orbital component of the wave function can be represented as $0.998(10)^1_1$ for the 2^- state and $0.688(10)^0_1 + 0.309(10)^1_1$ for the 1^- state. Here, the LS-coupling scheme is presented by the following form $(l\lambda)^S_L$ with the definitions: l is orbital momentum of the core nucleus (^{5}He), λ is orbital momentum of the hyperon, and S and L are the total spin and total orbital momentum. Thus, the state 2^- is pure triplet spin state and state 1^- is superposition of triplet and singlet spin states. Note, however, that this calculation underestimated the binding energy of $^6_\Lambda$He for the potentials chosen. In Ref. [3] a similar treatment was performed with modified potentials. This calculation overestimated the system. A new corrected value for the binding energy of $^6_\Lambda$He was reported in Ref. [9]. The value is within the experimental errors, but slightly overestimates the system.

In the present work, the $^6_\Lambda$He hypernuclus is modeled as a cluster $\alpha\Lambda n$ system. We use the potentials proposed for the αn and Λn interactions in Refs. [10, 11] to evaluate energies of the $(1^-, 2^-)$ spin doublet. The model does not include the ΛN–ΣN coupling that may be strong, as it was indicated in [12]. Our calculations are based on the configuration-space Faddeev equations for a system of three non-identical particles. We compare our numerical results with those of the [3, 9] calculations and with the experimental data.

2. Model

The $\alpha\Lambda n$ cluster system may be described by the Faddeev equations [13] which are given as follows

$$\begin{aligned}
(H_0 + V_{\Lambda n} - E)U_1 &= V_{\Lambda n}(U_2 + U_3), \\
(H_0 + V_{\alpha n} - E)U_2 &= V_{\alpha n}(U_1 + U_3), \\
(H_0 + V_{\alpha \Lambda} - E)U_3 &= V_{\alpha \Lambda}(U_1 + U_2).
\end{aligned} \tag{1}$$

The total wave function of the system Ψ is decomposed in the sum of the Faddeev components U_1, U_2, and U_3 corresponding to the $(\Lambda n)\alpha$, $(\alpha n)n$, and $(\alpha \Lambda)n$ types

of rearrangements, respectively:

$$\Psi = U_1 + U_1 + U_3.$$

$V_{\Lambda n}$, $V_{\alpha n}$, and $V_{\alpha \Lambda}$ are pair potentials of interactions between $\Lambda - n$, $\alpha - n$, and $\alpha - \Lambda$.

The differential form of the Faddeev equations means that each Faddeev component is expressed in its own Jacobi coordinates $\mathbf{x}_i, \mathbf{y}_i$, $i=1,2,3$

$$U_i = U_i(\mathbf{x}_i, \mathbf{y}_i).$$

In Eq. (1), H_0 is differential operator $H_0 = -\frac{\hbar^2}{m}(\Delta_{\mathbf{x}_i} + \Delta_{\mathbf{y}_i})$. The Jacobi vectors are linearly related by the orthogonal transformation

$$\left(\begin{array}{c} \mathbf{x_i} \\ \mathbf{y_i} \end{array} \right) = \left(\begin{array}{cc} C_{ij} & S_{ij} \\ -S_{ij} & C_{ij} \end{array} \right) \left(\begin{array}{c} \mathbf{x_j} \\ \mathbf{y_j} \end{array} \right) , \quad C_{ij}^2 + S_{ij}^2 = 1,$$

where

$$C_{ij} = -\sqrt{\frac{m_i m_j}{(M - m_i)(M - m_j)}} \ , \quad S_{ij} = (-)^{j-i}\text{sign}(j - i)\sqrt{1 - C_{ij}^2}.$$

M is the total mass of the system and m_i is the mass of the i-th particle.

The LS-coupling scheme is used for partial wave analysis of the equations (1). A quantum state in LS basis is defined by a set of quantum numbers $\{(l\lambda)L(\sigma s)S\} \equiv \alpha$. In this basis the spin-angular eigenfunctions have the form

$$W_\alpha(\hat{\mathbf{x}}, \hat{\mathbf{y}}) = < \hat{\mathbf{x}}, \hat{\mathbf{y}}|\alpha > = \left[[Y_l(\hat{\mathbf{x}}) \otimes Y_\lambda(\hat{\mathbf{y}})]^{LL_z} \otimes [\sigma \otimes s]^{SS_z} \right]^{J,J_z}.$$

For convenience, one may use the bipolar harmonics:

$$\mathcal{Y}_{l\lambda}^{LL_z}(\hat{\mathbf{x}}, \hat{\mathbf{y}}) = [Y_l(\hat{\mathbf{x}}) \otimes Y_\lambda(\hat{\mathbf{y}})]^{LL_z} = \sum_{m_l, m_\lambda} C_{lm_l \lambda m_\lambda}^{LL_z} Y_{lm_l}(\hat{\mathbf{x}}) Y_{\lambda m_\lambda}(\hat{\mathbf{y}}).$$

The explicit form of the eigenfunction is

$$W_\alpha(\hat{\mathbf{x}}, \hat{\mathbf{y}}) = \sum_{L_z, S_z, \sigma_z, s_z} C_{LL_z SS_z}^{JJ_z} C_{\sigma \sigma_z s s_z}^{SS_z} \mathcal{Y}_{l\lambda}^{LL_z}(\hat{\mathbf{x}}, \hat{\mathbf{y}}) \chi_{\sigma_z}^{\sigma} \chi_{s_z}^{s}.$$

Correspondingly the Faddeev component expanded in the basis $|\alpha >$ is

$$U_i(\mathbf{x_i}, \mathbf{y_i}) = \sum_\alpha \frac{\Phi_\alpha(x_i, y_i)}{x_i, y_i} W_\alpha(\hat{\mathbf{y_i}}, \hat{\mathbf{x_i}}).$$

The sum over $\alpha = \{(l\lambda)L(\sigma s)S\}$ is restricted by the coupling triangular condition and the parity conservation condition $(-1)^{\lambda+l} = (-1)^\pi$. The final set of the equations for the Faddeev components (1) is

$$\left[-\frac{\hbar^2}{m}(\partial_{x_i^2} + \partial_{y_i^2}) + v_\alpha^{cen.}(x_i, y_i) - E) \right] \Phi_\alpha(x_i, y_i) = -\sum_\beta v_{\alpha\beta}(x_i) \left[\Phi_\beta(x_i, y_i) \right.$$

$$+\frac{1}{2}\int_{-1}^{1}du\sum_{\gamma}\Big(\frac{x_iy_i}{x_jy_j}h_{\beta\gamma}(x_j,y_j,u)\Phi_{\gamma}(x_j,y_j)+\frac{x_iy_i}{x_ky_k}h_{\beta\gamma}(x_k,y_k,u)\Phi_{\gamma}(x_k,y_k)\Big)\Big], \quad i\neq j\neq k,$$

$$(2)$$

where the centrifugal potential is

$$v_{\alpha}^{cen.}(x_i,y_i)=\frac{\hbar^2}{m}\Big[\frac{l(l+1)}{x_i^2}+\frac{\lambda(\lambda+1)}{y_i^2}\Big].$$

The $v_{\alpha\beta}(x_i)$ are matrix elements of nuclear potentials. General form of the $h_{\alpha'\alpha}(x_j,y_j,u)$ functions has the form

$$h_{\alpha'\alpha}(x_j,y_j,u)=(-)^{l'+\lambda'+L}\prod_{l\lambda}{}^{2}\prod_{l'\lambda'}(2l)!(2\lambda)!\sum_{\substack{l_1+l_2=l\\\lambda_1+\lambda_2=\lambda}}(-)^{\lambda_1}\frac{x_i^{l_1+\lambda_1}y_i^{l_2+\lambda_2}}{x_j^ly_j^{\lambda}}$$

$$\frac{C_{ji}^{l_1+\lambda_2}S_{ji}^{l_2+\lambda_1}}{\sqrt{(2l_1)!(2l_2)!(2\lambda_1)!(2\lambda_2)!}}\sum_{k}(-)^k(2k+1)P_k(u)\sum_{l''\lambda''}C_{l_10\lambda_10}^{l''0}C_{l_20\lambda_20}^{\lambda''0}C_{l'0k0}^{l''0}C_{\lambda'0k0}^{\lambda''0}$$

$$\left\{\begin{array}{ccc}l'&l''&k\\\lambda''&\lambda'&L\end{array}\right\}\left\{\begin{array}{ccc}l_1&\lambda_1&l''\\l_2&\lambda_2&\lambda''\\l&\lambda&L\end{array}\right\},$$

where $\prod_{l\lambda}=\sqrt{(2l+1)(2\lambda+1)}$ and $0\leq k\leq(\lambda''+l''+\lambda+l)/2$.

In the model, the potential αn is the only one that is written as a sum of the central and the spin-orbit parts. The matrix elements of the spin-orbit potential $V_{\alpha n}^{so}$ are given as

$$V_{\alpha n}^{so}(x)=\frac{2L+1}{2}\sum_{j=l\pm1/2}(2j+1)\left\{\begin{array}{ccc}J&L&1/2\\l&j&\lambda\end{array}\right\}^2$$

$$\times(j(j+1)-l(l+1)-3/4)v_{so}(x),$$

where $v_{so}(x)$ is a coordinate part of the αn spin-orbit potential. In the LS basis the αn potential is represented by diagonal matrix with diagonal elements: $v_{\alpha n}^l(x)=v_c^l(x)+V_{\alpha n}^{so}(x)$, where $v_c^l(x)$ is central l-wave partial component of the αn potential.

We consider spin-orbital configurations in the system corresponding to the total angular momentum J^{π} equal to 1^- and 2^-. The LS basis allows to restrict 2^- model space to the states with the total spin $S=1$ (when the spin projections of hyperon and nucleon are parallel). For 1^- state, the possible spin-orbital momentum configurations are represented with a set of $S=0$ and $S=1$.

For both considered states 1^- and 2^-, the total orbital momentum $L^{\pi}=1^-$ has the following combinations of the subsystems orbital momenta:

$$(\ell_{\alpha\Lambda},\lambda_{(\alpha\Lambda)-n})=(0,1),(1,0),(1,2),$$

$$(\ell_{\alpha n},\lambda_{(\alpha n)-\Lambda})=(0,1),(1,0),(1,2),(2,1),$$

$$(\ell_{\Lambda n},\lambda_{(\Lambda n)-\alpha})=(0,1).$$

Table 1: Parameters of the αn and $\alpha\Lambda$ potentials (3). The pair angular momentum is l. V_{att}^l (V_{rep}^l) and V_c are given in MeV, β_{att}^l (β_{rep}^l) in fm^{-1}, and α_c in fm^{-2}.

Potential	Component	l	V_{rep}^l	β_{rep}^l	V_{att}^l	β_{att}^l
αn	central	0	50.0	1/2.3	–	–
		1	40.0	1/1.67	63.0	1/2.3
		2	–	–	21.93	1/2.03
	spin-orbit	–	–	–	38.0	1/1.67
$\alpha\Lambda$	central	0	–	–	54.36	0.538

Within our model, the αn interaction is constructed to reproduce the results of R-matrix analysis for $\alpha-$n scattering data [14]. This potential simulates the Pauli exclusion principle for αn in the s-state with repulsive core. This new αn potential was proposed in Ref. [10]. The parameters of the potential were obtained by modification of the potential given in Ref. [15]. Using this potential, we have modeled the ^{6}He nucleus as the αnn cluster system. The results of the calculations reproduce well the experimental data for the low-lying spectrum of the nucleus [16]. The potential is represented as one and two range Gaussian function:

$$V^l(x) = V_{rep}^l \exp(-\beta_{rep}^l x)^2 - V_{att}^l \exp(-\beta_{att}^l x)^2 \ . \tag{3}$$

The parameters of the potential are presented in Tab. 1.

An $\alpha\Lambda$ potential was proposed in Ref. [17] to describe the $^9_\Lambda$Be hypernucleus within the $\alpha\alpha\Lambda$ cluster model. The parameters of the potential are given in Tab. 1. The chosen parameters allowed us to reproduce the energy of the ground state and the first excited state of $^9_\Lambda$Be, simultaneously [18].

We use the ΛN potential simulating the NSC97 model of barion-barion interaction [19]. This s-wave potential is the three-range Gaussian [20] corrected in [11] and has the following form:

$$V^{(2S+1)}(x) = \sum_i^3 v_i^{(2S+1)} \exp(-\frac{x^2}{\beta_i^2}) \ . \tag{4}$$

Here S is spin of the pair (S=0 or 1). The values of the range parameters β_i and of the singlet- and triplet-strength parameters $v_i^{(2S+1)}$ are listed in table 2. In this table the parameter γ was chosen so that the potential (4) might reproduce the scattering length and the effective range for the given model [19]. In particular, singlet (triplet) scattering length is -2.5 fm (-1.75 fm).

3. Results of calculations

The states 1^- and 2^- of the cluster $\alpha\Lambda$n system are close to the two body $^5_\Lambda$He+n threshold. The lower level of the doublet (1^-, 2^-) is a bound state. The upper member is a resonance state. To evaluate energies of these states, we solved

Table 2: Parameters of the Λn potential (4). The NSC97f model of barion-barion interaction is simulated by choosing $\gamma^{(1)}=1.0581$ and $\gamma^{(3)}=1.0499$ [11].

i	β_i (fm)	$v_i^{(1)}$ (MeV)	$v_i^{(3)}$ (MeV)
1	1.342	-21.49	-21.39
2	0.777	$-379.1 \times \gamma^{(1)}$	$-379.1 \times \gamma^{(3)}$
3	0.350	9324	11359

numerically the bound state problem formulated by Eq. (2), applying the finite difference approximation for radial variable and the spline collocation method for angular one. Resulted matrix eigenvalue problem has been solved with the inverse iteration method. For the resonance state, the method of analytical continuation in a coupling constant is used to calculate parameters of the resonance [21]. The coupling constant is the depth of an additional non-physical three-body potential [23, 22]. The potential has the form: $V_3(\rho) = -\delta \exp(-\alpha\rho^2)$ with parameters α, $\delta \geq 0$ that can be varied. For each resonance there exists a region $|\delta| > |\delta_0|$ where a resonance becomes a bound state. In this region we calculated $2N$ bound state energies corresponding to $2N$ values of δ. The continuation of this energy set as a function of δ onto complex plane is carried out by means of the Páde approximant: $-\sqrt{E} = \sum_{i=1}^{N} p_i \zeta^i / (1 + \sum_{i=1}^{n} q_i \zeta^i)$, where $\zeta = \sqrt{\delta - \delta_0}$. The complex value
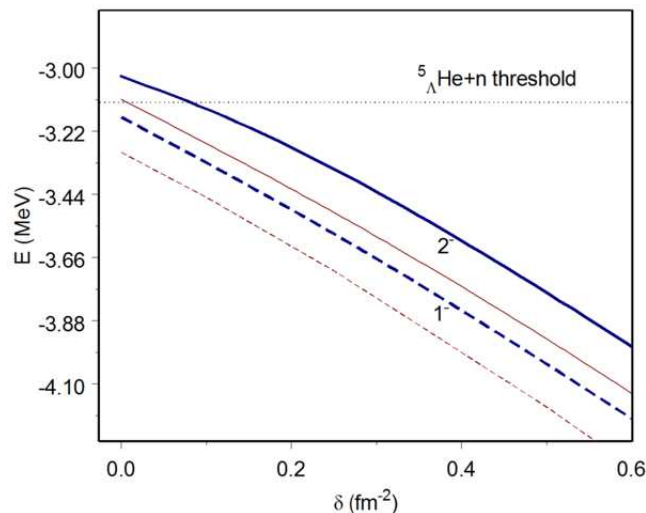

Figure 1: Real part of the Páde approximant for 2^- state (solid line) of the $\alpha+\Lambda+n$ system. Calculated resonance energy correspons to the Páde approximant value for $\delta=0$. Energy is measured from the $\alpha + n + n$ threshold. The calculations for 1^- state are presented by dashed line. The range parameter α for used the three-body potential is 0.3 fm^{-2}. Tge results that correspond to the calculations with the three-body force $V_{3bf}(\rho) = V_0 \exp(-\alpha_0\rho^2)$ are shown by the fine lines. The potential has parameters V_0=-0.05 fm^{-2} and α_0=0.2 fm^{-2}.

of the Páde approximant for δ=0 gives the energy and width of the resonance: $E(\delta = 0) = E_r + i\frac{\Gamma}{2}$.

Results of the calculation are shown in Fig. 1. The calculated value for hyperon binding energy of $^{6}_{\Lambda}$He(1^{-}) is 0.65 MeV. The resonance energy of $^{6}_{\Lambda}$He(2^{-}) is about 0.11 MeV. Since the experimental value for the $^{6}_{\Lambda}$He ground state was reported to be E_{Λ}=-0.17$\pm$ 0.10 MeV, our cluster model underestimates the hypernucleus. It is not surprising, taking into account the fact that the similar αnn cluster system is also weakly bound (or is not bound) for the pair potentials under consideration. The cluster structure of such systems may be distorted at small distances between particles. Use of the three-body force is proposed as an acceptable solution of the problem. Parameters of the three-body potential can be chosen to reproduce experimental data (for instance, see our work [16]). The three-body force makes an equidistant energy shift of spectral levels [16] (see also Fig. 1). There could be a total orbital momentum dependence [16] of the three-body potential. However, for the considered case of the doublet states (1^{-}, 2^{-}) of the $\alpha\Lambda$n system, this dependence may be ignored due to the same orbital momentum for both states. The three-body potential defined as $V_{3bf}(\rho) = V_0 \exp(-\alpha_0 \rho^2)$ with the parameters V_0=-0.05 fm^{-2} and α_0=0.2 fm^{-2} allowed us to reproduce the experimental value for the hyperon ground state energy, to be about 0.17$\pm$ MeV. The energy of the 2^{-} resonance becomes close to the $^{5}_{\Lambda}$He+n threshold. This situation is shown in Fig. 1. Our prediction for the E_x=0.18 MeV for (1^{-},2^{-}) energy spacing can be directly compared with either experimental data (not measured yet) or other calculations. In Fig. 2 we present our result together with the results of calculations from [3] and [9]. In the model [9], a potential simulating Nijmegen model NSC97f was used for the Λn interaction. The calculations resulted in the hyperon separation energy B_{Λ}=4.21 MeV, which is close to the observed B_{Λ}=4.18$\pm$0.10 MeV. Our results for (1^{-},2^{-}) energy spacing differ from that obtained within the model of Ref. [9]. At the same time, 2^{-} resonance energies are similar.
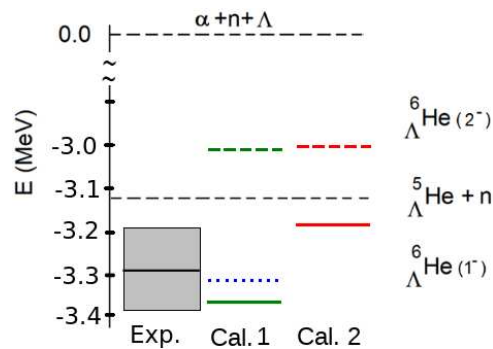

Figure 2: Spin-doublet (1^{-},2^{-}) of the $^{6}_{\Lambda}$He nucleus. Cal. 1 corresponds to the Refs. [3] and [9]. Cal. 2 are the present calculations. Exp. represents the experimental data with the error bar. Solid (dashed) lines are results calculated for the 1^{-} bound state (2^{-} resonance). Doted line is the result for 1^{-} state from Ref. [9].

4. Conclusions

The three-body $\alpha\Lambda n$ cluster models with the OBE simulating (NSC97f) potential for the Λn interaction was applied for calculation of $(1^-,2^-)$ spin doublet energies of the $^6_\Lambda$He hypernucleus. We obtained the value of 0.18 MeV for the $(1^-,2^-)$ energy spacing. The calculated 2^- excitation energy is in a good agreement with the results of another calculations whereas 1^- binding energy disagrees with that calculations. The obtained results for the bound state energy is lower than the experimental value. It may be an evidence for the violation of the exact three-body cluster structure of the system. The three-body force is required to describe experimental data within this cluster model.

References

[1] Hashimoto O., Tamura H., *Spectroscopy of Λ hypernuclei*. Progress in Particle and Nuclear Physics 2006, **57**, pp. 564–653

[2] Motoba T., Bando H. and Ikeda K., *Light p-Shell Λ-Hypernuclei by the Microscopic Three-Cluster Model*. Prog. Theor. Phys. 1983, **70**, pp. 189–221

[3] Hiyama E., Kamimura M., Motoba T., Yamada T. and Yamamoto Y., *Three-body model study of A=6–7 hypernuclei: Halo and skin structures*. Phys. Rev. C 1996, **53**, pp. 2075–2084; Hiyama E., Kamimura M., Yamamoto Y., Motoba T. and Rijken Th.A., *S = −1 Hypernuclear Structure*. Prog. Theor. Phys. Supplement 2010, **185**, pp. 152–196

[4] Nakamura S.N. et al., *Observation of the $^7_\Lambda$He hypernucleus by the $(e, e'K^+)$ reaction*. Phys. Rev. Lett. 2013, **110**, 012502, 5pp.

[5] Agnello M., et al., *Evidence for Heavy Hyperhydrogen $^6_\Lambda$H*. Phys. Rev. Lett. 2012, **108**, 042501, 5pp.

[6] Botvina A.S., Mishustin I.N., Pochodzalla J., *Production of exotic hypernuclei from excited nuclear systems*. Phys. Rev. C 2012, **86**, pp. 011601(R), 5pp.

[7] Davis D.H., *50 years of hypernuclear physics I. The early experiments*. Nucl. Phys. A 2005, **754**, pp. 3c–13c; Esser A. et al. *Prospects for hypernuclear physics at Mainz: From KAOS@MAMI to PANDA@FAIR*. Nucl. Phys. A 2013, **914**, pp.519–529

[8] Filikhin I., Suslov V.M. and Vlahovic B., *A new prediction for binding energy of the $^7_\Lambda$He hypernucleus*. J. Phys. G: Nucl. Part. Phys. 2005, **31**, pp. 389-400

[9] Hiyama E., Ohnishi S., Kamimura M., Yamamoto Y., *Four-body structure of neutron-rich hypernucleus $^6_\Lambda$H*. Nucl. Phys. A 2013, **908**, pp. 29–39

[10] Filikhin I., Suslov V.M. and Vlahovic B. ^9Be *Low-Lying Spectrum within a Three-Cluster Model.* Few-Body Syst. 2011, **50**, pp. 255–257; Filikhin I., Suslov V.M. and Vlahovic B., *Spin-flip doublets of* ^9Be *spectrum within cluster model.* FIZIKA B (Zagreb) 2011, **20**, 3, pp. 18–202

[11] Filikhin I., Gal A., *Light* $\Lambda\Lambda$ *hypernuclei and the onset of stability for* $\Lambda\Xi$ *hypernuclei.* Phys. Rev. C 2002, **65**, 041001R, 4pp.; Filikhin I. and Gal A., *Faddeev-Yakubovsky Search for* $^4_{\Lambda\Lambda}H$. Phys. Rev. Lett. 2002, **89**, 172502, 4pp.

[12] Millener D.J., *Hypernuclear Gamma-Ray Spectroscopy and the Structure of p-shell Nuclei and Hypernuclei.* Lect. Notes Phys. 2007, **724**, pp. 31–81; arXiv:0902.2142v1

[13] Faddeev L.D. and Merkuriev S.P., *Quantum Scattering Theory for Several Particle Systems* (Kluwer Academic, Dordrecht, 1993), 398pp.

[14] Nollett K.M., Pieper S.C., Wiringa R.B., Carlson J. and Hale G.M., *Quantum Monte Carlo Calculations of Neutron-α Scattering.* Phys. Rev. Lett. 2007, **99**, 022502, 4pp.

[15] Cobis A., Fedorov D.V., Jensen A.S., *Three-body halos. V. Computations of continuum spectra for Borromean nuclei.* Phys. Rev. C 1998, **58**, pp. 1403–1421

[16] Filikhin I., Suslov V.M. and Vlahovic B., *A cluster calculation for* ^6He *spectrum*, will be appeared in Phys. Atom. Nucl. 2014 (DOI: 10.7868/S0044002714030088)

[17] Filikhin I., Suslov V.M. and Vlahovic B., *Faddeev calculations for the* $^9_{\Lambda}Be$ *spectrum.* EPJ Web of Conferences 2010, **3**, 07004, 7pp.

[18] Filikhin I., Suslov V.M. and Vlahovic B., *An α-Cluster model for* $^9_{\Lambda}Be$ *spectroscopy.* Phys. Atom. Nucl. 2013, **76**, pp. 355–364

[19] Rijken Th.A., Stoks V. G.J. and Yamamoto Y., *Soft-core hyperon-nucleon potentials.* Phys. Rev. C 1999, **59** pp. 21-40

[20] Hiyama E., Kamimura M., Motoba T., Yamada T. and Yamamoto Y., *Three- and Four-Body Cluster Models of Hypernuclei Using theG-Matrix ΛN Interaction* $^9_{\Lambda}Be$, $^{13}_{\Lambda}C$, $^6_{\Lambda\Lambda}He$ *and* $^{10}_{\Lambda\Lambda}Be$. Prog. Theor. Phys. 1997, **97** pp. 881–899

[21] Kukulin V.I., Krasnopolsky V.M. and Horacek J., *Theory of Resonances* (Kluwer, Dordrecht 1989).

[22] Filikhin I., Suslov V.M. and Vlahovic B., 0^+ *states of the* ^{12}C *nucleus: the Faddeev calculation in configuration space.* J. Phys. G: Nucl. Part. Phys. 2005, **31**, pp. 1207–1224

[23] Kurokawa C. and Kato K., *New broad 0^+ state in* ^{12}C. Phys. Rev. C 2005, **71**, 021301(R), 4pp.