http://mmg.tversu.ru ISSN 2311-1275

Mathematical Modelling and Geometry

Volume 1, No 3, p. 1 – 12 (2013)

Extension of the zero-range potential model onto the Hamiltonians with a singularity at the origin

S. L. Yakovlev[a], **V. A. Gradusov**[b]

Department of Computational Physics, St Petersburg State University, 198504, St Petersburg, Russia

e-mail: [a] yakovlev@cph10.phys.spbu.ru, [b] vitaly.gradusov@gmail.com

Abstract. We evaluate the short-range asymptotic behavior of the Green function for a Hamiltonian when its potential energy part has an inverse power singularity at the origin. The analytically solvable case of sharply screened Coulomb potential is considered firstly. For this potential the additional logarithmic singular term has been found in the short-range asymptote of the Green function as in the case of the pure Coulomb potential. The case of a short-range potential of an arbitrary form with inverse power singularity is treated on the basis of the integral Lippmann-Schwinger equation. It is shown that, if the singularity is weaker than the Coulomb one, the Green function has only standard singularity. For the case of $r^{-\rho}$ singularity of the potential with $1 \leq \rho < 2$ the additional singularity in the asymptotic behavior of the Green function appears. In the case of $\rho = 1$ the additional logarithmic singularity has the same form as in the case of the pure Coulomb potential. In the case of $1 < \rho < 2$ the additional singularity of the Green function has the form of the polar singularity $r^{-\rho+1}$. These results are applied for extending the zero-range potential formalism on Hamiltonians with singular potentials.

Keywords: Green function, sharply screened Coulomb potential, exact solutions, zero-range potential

PACS numbers: 03.65.Nk, 34.80.-Bm

References

[1] Albeverio S., Gesztesy F., Hoegh-Krohn R., Holden H., *Solvable Models in Quantum Mechanics.* 2005, AMS Chelsea Publishing: Providence, RI

[2] Yakovlev S.L., Gradusov V.A. *Zero-range potential for particles interacting via Coulomb potential.* J. Phys. A 2012, **46**, 035307, 15pp.

[3] Yakovlev S.L., Volkov M.V., Yarevsky E., Elander N. *The impact of sharp screening on the Coulomb scattering problem in three dimensions.* J. Phys. A 2010, **43**, 245302, 14pp.

[4] Newton R.G. *Scattering Theory of Waves and Particles.* 1982, Springer-Verlag: New York

[5] Titchmarsh E.C. *Eigenfunction Expansions Associated with Second-order Differential Equations. Part 1.* 1962, Oxford at the Clarendon Press: London

[6] Messiah A. *Quantum Mechanics. Vol. 1.* 1962, J Willey and Sons, Inc.: New-York

[7] Hostler L. *Coulomb Green's Functions and the Furry Approximation.* J. Math. Phys. 1964, **5**, pp. 591-611

[8] Abramowitz M., Stegun I. *Handbook of Mathematical Functions.* 1964, National Bureau of Standards: Washington, D.C.

[9] Povzner A.Ya. *On the expansion of arbitrary functions in characteristic functions of the operator* $-\Delta u + cu$. Matem. Sbornik 1953, **32(74)**, pp. 109-156