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Expressions of players' guaranteed payoffs in a noncooperative game modelling commodity purchase and realization process

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Abstract. A noncooperative game that models a process of purchase and realization of a certain commodity is considered. The participants of the modelled process are two shops competing with each other. The players in the considered game correspond to these shops. Expressions of players' guaranteed payoffs and also of upper evaluations of the guaranteed payoffs are determined. The determination of these expressions is factually reduced to the investigation of properties of some surfaces.

Keywords: game, noncooperative game, player, strategy, situation, payoff, payoff function, guaranteed payoff, shop, commodity, purchase, sale, price, buyer, profit
MSC numbers: 91A10

1. Introduction

In this paper we use a noncooperative game [1, 2, 3, 4, 5, 6, 7, 8] as a mathematical model of the investigated process. Processes described by noncooperative games are characterized by the following points:

1) n parties participate in the process,

2) each of the parties is to single out a solution out of a given variety for the party,

3) for each party there is defined a mapping that associates to an arbitrary set of solutions selected by the participants of the process a number having a sense of usefulness of the set for the party.

Parties participating in the process are called 'players'. Solutions chosen by this or that player are called 'strategies of the player'. Sets of strategies selected by the players are called 'situations'. A mapping that associates to an arbitrary situation a number having a sense of usefulness of the situation for a player is called 'a payoff function of the player', and the number itself – 'the player's payoff' according to the situation. A mapping that associates to an arbitrary strategy of the player the largest of the numbers (the player's payoff cannot be less than these numbers providing the player has chosen the specified strategy) is called 'the player's guaranteed payoff'.

2. Description of the investigated game

A noncooperative game describing a process of purchase and realization of a certain commodity is formed in [9]. The specified process consists in the following. Two shops (Shop 1 and Shop 2) buy a certain commodity at the price γ ($\gamma > 0$) at some warehouse. At the moment of purchase the i-th shop, $i \in \{1, 2\}$, has got money funds in amount of $s(i)$ ($s(i) > 0$). It is supposed that the quantity of the commodity available at the warehouse at the moment would be enough for each shop to accomplish the purchase of the commodity at the price of γ on all the money available at the shop. Before the moment of purchase the i-th shop, $i \in \{1, 2\}$, determines the values of the following quantities:

$t(i)$ – the quantity of the commodity bought by the i-th shop and

$c(i)$ – the price at which the commodity will be sold at the i-th shop,

$t(i) \in [0, s(i)/\gamma]$ and $c(i) > 0$. Assume $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$. It is supposed that if $c(i) < c(j)$, i.e. if the price of the commodity in the i-th shop is lower than in the j-th shop, then if the commodity is available in the both shops, then buyers prefer the i-th shop that is the shop where the commodity is cheaper. But if $c(i) = c(j)$, i.e. if the commodity is sold in the both shops at the same price, then if the commodity is available in the both shops, the speed of its purchase is the same at the both shops. The total demand for the commodity is m units ($m > 0$), and the money that buyers allocate to the commodity purchase is d units ($d > 0$). It is assumed that altogether there are N buyers ($N \geqslant 1$) and that the demand of each buyer is m/N and the money allocated by an arbitrary buyer to the commodity purchase is d/N. The commodity is realized by the shops for a certain period of time, and the part of it unsold at the end of this period is returned to the warehouse at the price λ ($\lambda \in (0, \gamma)$). The aim of each shop is the maximization of the profit obtained from the commodity realization.

The noncooperative game [9] describing this process is the set

$$\Gamma \;=\; \langle I, V(1), V(2), J(1), J(2)\rangle.$$

In this set I is a set of numbers of players. $I = \{1, 2\}$. The i-th player, $i \in I$, corresponds to the i-th shop.

$V(i)$, $i \in I$, is a set of strategies of the i-th player. $V(i) = [0, s(i)/\gamma] \times (0, +\infty)$, $i \in I$. An arbitrary strategy $v(i)$ of the i-th player, $i \in I$, is a pair $(t(i),c(i))$ of values defined by the i-th shop before the moment of the commodity purchase. These values are pointed out above.

$J(i)$, $i \in I$, is a payoff function of the i-th player. The domain of the function $J(i)$, $i \in I$, is a set V of situations ($V = V(1) \times V(2)$), the function values belong to Set $\mathbf{R}$; the given function associates to an arbitrary situation v from Set V the value of the profit obtained by the i-th shop from the commodity realization; this value turns out to be equal to

$$c(i) \cdot \tau(i)(v) \;+\; \lambda \cdot (t(i) - \tau(i)(v)) \;-\; \gamma \cdot t(i)$$

here $\tau(i)(v)$ is a value (corresponding to situation v) of the function $\tau(i)$. The domain of function $\tau(i)$ is Set V, the function values belong to Set $\mathbf{R}$; function $\tau(i)$ associates to an arbitrary situation v from Set V the quantity of the commodity sold in the i-th shop to buyers (during the whole period of realization).

Assume v is an arbitrary situation from Set V and i is an arbitrary element from Set I. Further on the i-th component of situation v i.e. the strategy of the i-th player in the situation will be denoted by the symbol $v(i)$.

Assume i is an arbitrary element from Set I and $v(i)$ is an arbitrary strategy of the i-th player. Further on the first and the second components of the strategy $v(i)$ will be denoted by the symbols $t(i)$ and $c(i)$ respectively.

Let $i \in I$, $j \in I \setminus \{i\}$ and $v \in V$. In [9] it is ascertained that the following relations are true

1) $\tau(i)(v) \;=\; \min\{m, d/c(i), t(i)\},$
   if $c(i) \;<\; c(j),$

2) $\tau(i)(v) \;=\; 0,$
   if $(c(i) \;>\; c(j)) \;\wedge\; (\min\{m, d/c(j)\} \;\leqslant\; t(j)),$

3) $\tau(i)(v) = \min\{(d - c(j) \cdot t(j))/c(i), t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < d/c(j) < m)$,

4) $\tau(i)(v) = \min\{m - t(j), t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < m \leqslant d/c(j)) \wedge (d/c(i) \geqslant m)$,

5) $\tau(i)(v) = \min\{d \cdot (m - t(j))/m/c(i), t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < m = d/c(j)) \wedge (d/c(i) < m)$,

6) $\tau(i)(v) = \min\{d \cdot (m - t(j))/m/c(i) + \Delta, t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < m < d/c(j)) \wedge (d/c(i) < m)$
(here $\Delta = \min\{(1 - q) \cdot (c(i) \cdot m - d), q \cdot (d - c(j) \cdot m)\}/N/c(i)$
where $q \in [0, 1)$),

7) $\tau(i)(v) = \min\{m/2, d/2/c(i)\}$,
if $(c(i) = c(j)) \wedge (\min\{m/2, d/2/c(i)\} \leqslant \min\{t(i), t(j)\})$,

8) $\tau(i)(v) = t(i)$,
if $(c(i) = c(j)) \wedge (\min\{m/2, d/2/c(i)\} > t(i)) \wedge (t(i) \leqslant t(j))$,

9) $\tau(i)(v) = \min\{m - t(j), t(i)\}$,
if $(c(i) = c(j)) \wedge (t(i) > t(j)) \wedge (t(j) < m/2 \leqslant d/2/c(i))$,

10) $\tau(i)(v) = \min\{d/c(i) - t(j), t(i)\}$,
if $(c(i) = c(j)) \wedge (t(i) > t(j)) \wedge (t(j) < d/2/c(i) < m/2)$.

Let's make the following change in Game Γ: assume now that $\tau(i)$, $i \in I$, is a function with the domain V and a set of values within $\mathbf{R}$ that associates to an arbitrary situation v the value equal to

1) $\min\{m, d/c(i), t(i)\}$,
if $c(i) < c(j)$,

2) 0,
if $(c(i) > c(j)) \wedge (\min\{m, d/c(j)\} \leqslant t(j))$,

3) $\min\{(d - c(j) \cdot t(j))/c(i), t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < d/c(j) < m)$,

4) $\min\{m - t(j), t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < m \leqslant d/c(j)) \wedge (d/c(i) \geqslant m)$,

5) $\min\{d \cdot (m - t(j))/m/c(i), t(i)\}$,
if $(c(i) > c(j)) \wedge (t(j) < m \leqslant d/c(j)) \wedge (d/c(i) < m)$,

6) $\min\{m/2, d/2/c(i)\}$,
if $(c(i) = c(j)) \wedge (\min\{m/2, d/2/c(i)\} \leqslant \min\{t(i), t(j)\})$,

7) $t(i)$,
if $(c(i) = c(j)) \wedge (\min\{m/2, d/2/c(i)\} > t(i)) \wedge (t(i) \leqslant t(j))$,

8) $\min\{m - t(j), t(i)\}$,
if $(c(i) = c(j)) \wedge (t(i) > t(j)) \wedge (t(j) < m/2 \leqslant d/2/c(i))$,

9) $\min\{d/c(i) - t(j), t(i)\}$,
if $(c(i) = c(j)) \wedge (t(i) > t(j)) \wedge (t(j) < d/2/c(i) < m/2)$.

3. Determination of players' guaranteed payoffs expressions in the investigated game

Assume $i \in I$ and $j \in I \setminus \{i\}$. Further on to denote an arbitrary situation v the following record will also be used

$$v(i); v(j)$$

The guaranteed payoff of the i-th player, $i \in I$, i.e. the function with the domain $V(i)$ and a set of values within $\mathbf{R} \cup \{-\infty\}$ that associates to an arbitrary strategy $v(i)$ of the i-th player the value equal to

$$\inf_{v(j) \in V(j)} J(i)(v(i); v(j))$$

will be denoted by the symbol $G(i)$.

Assume i is an arbitrary element from Set I. The function with the domain $V(i)$ and a set of values within $\mathbf{R}$ that associates to an arbitrary strategy $v(i)$ of the i-th player the value equal to

$$\min\{m, d/c(i)\}$$

will be denoted by the symbol $\mu(i)$.

Further in the work a number of propositions is given for Game Γ. In the last proposition the final result of the work is laid down. In this proposition expressions of players' guaranteed payoffs and also of upper evaluations of the guaranteed payoffs in Game Γ are given.

Proposition 1.

Let $i \in I$.

The following holds

$$\forall v \in V, \tau(i)(v) \geqslant 0.$$

To prove Proposition 1 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 2.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds: the value

$$\inf_{v(j) \in V(j)} \tau(i)(v(i); v(j))$$

is a finite non-negative number.

To prove Proposition 2 we use Proposition 1 and the definition of inf.

Proposition 3.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

$$
\begin{array}{llll}
\textit{1)} & G(i)(v(i)) & = & s(i), \textit{ if } c(i) \in (0,\lambda) \textit{ and } t(i) = 0, \\
\textit{2)} & G(i)(v(i)) & \leqslant & s(i) - (\gamma - \lambda)\cdot t(i), \textit{ if } c(i) \in (0,\lambda) \quad \textit{and } t(i) \in (0, s(i)/\gamma], \\
\textit{3)} & G(i)(v(i)) & = & s(i) - (\gamma - \lambda)\cdot t(i), \textit{ if } c(i) = \lambda, \\
\textit{4)} & G(i)(v(i)) & = & s(i) - (\gamma - \lambda)\cdot t(i) + (c(i) - \lambda)\cdot \inf\limits_{v(j) \in V(j)} \tau(i)(v(i); v(j)), \\
 & & & \textit{if } c(i) \in (\lambda, +\infty).
\end{array}
$$

To prove Proposition 3 we use Propositions 1 and 2, the definitions of the functions $\tau(i)$, $J(i)$ and $G(i)$, $i \in I$, and the definition of inf.

Proposition 4.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds: the values

$$\inf_{v(j) \in [0, s(j)/\gamma] \times (0, c(i))} \tau(i)(v(i); v(j)),$$

$$\inf_{v(j) \in [0, s(j)/\gamma] \times \{c(i)\}} \tau(i)(v(i); v(j)) \quad \textit{and}$$

$$\inf_{v(j) \in [0, s(j)/\gamma] \times (c(i), +\infty)} \tau(i)(v(i); v(j))$$

are finite non-negative numbers.

To prove Proposition 4 we use Proposition 1 and the definition of inf.

Proposition 5.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

$$\inf_{v(j) \in V(j)} \tau(i)(v(i); v(j)) = \min \{ \inf_{v(j) \in [0, s(j)/\gamma] \times (0, c(i))} \tau(i)(v(i); v(j)),$$

$$\inf_{v(j) \in [0, s(j)/\gamma] \times \{c(i)\}} \tau(i)(v(i); v(j)),$$

$$\inf_{v(j) \in [0, s(j)/\gamma] \times (c(i), +\infty)} \tau(i)(v(i); v(j)) \}.$$

To prove Proposition 5 we use Proposition 4 and the definition of inf.

Proposition 6.

Let $i \in I$*,* $j \in I \setminus \{i\}$*,* $v(i) \in V(i)$*,* $c(j) \in (0, c(i))$*.*
The following holds: the value

$$\inf_{t(j) \in [0, s(j)/\gamma]} \tau(i)(v(i); (t(j), c(j)))$$

is a finite non-negative number.

To prove Proposition 6 we use Proposition 1 and the definition of inf.

Proposition 7.

Let $i \in I$*,* $j \in I \setminus \{i\}$*,* $v(i) \in V(i)$*.*
The following holds

$$\inf_{v(j) \in [0, s(j)/\gamma] \times (0, c(i))} \tau(i)(v(i); v(j)) = \inf_{c(j) \in (0, c(i))} \inf_{t(j) \in [0, s(j)/\gamma]} \tau(i)(v(i); (t(j), c(j))).$$

To prove Proposition 7 we use Proposition 6 and the definition of inf.

Proposition 8.

Assume some functions $f : X \to \mathbf{R}$ *and* $g : X \to \mathbf{R}$ *(here* X *is a nonempty set) answer the condition*

$$\forall x \in X, f(x) \leqslant g(x).$$

Then

$$\inf_{x \in X} f(x) \leqslant \inf_{x \in X} g(x).$$

To prove Proposition 8 we use the definition of inf.

Proposition 9.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $c(j) \in (0, c(i))$.

The following holds: function $f : [0, s(j)/\gamma] \to \mathbf{R}$ *defined with the help of the relationship*

$$f(t(j)) = \tau(i)(v(i); (t(j), c(j))), t(j) \in [0, s(j)/\gamma]$$

is a nonincreasing function.

To prove Proposition 9 we use Propositions 1 and 8 and the definition of the functions $\tau(i)$, $i \in I$.

Proposition 10.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

$$\inf_{v(j) \in [0, s(j)/\gamma] \times (0, c(i))} \tau(i)(v(i); v(j)) = \inf_{c(j) \in (0, c(i))} \tau(i)(v(i); (s(j)/\gamma, c(j))).$$

To prove Proposition 10 we use Propositions 7 and 9.

Proposition 11.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad m \leqslant s(j)/\gamma$.

Then

$$\tau(i)(v(i); (s(j)/\gamma, c(j))) = 0$$

with all $c(j) \in (0, c(i))$.

To prove Proposition 11 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 12.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad m \leqslant s(j)/\gamma$.

Then

$$\inf_{c(j) \in (0, c(i))} \tau(i)(v(i); (s(j)/\gamma, c(j))) = 0.$$

To prove Proposition 12 we use Proposition 11.

Proposition 13.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad m > s(j)/\gamma$, $\quad d/c(i) < s(j)/\gamma$.

Then

$$\tau(i)(v(i); (s(j)/\gamma, c(j))) =$$

$$= \begin{cases} \min\{d/c(i) \cdot (m - s(j)/\gamma)/m, t(i)\}, \text{ if } c(j) \in (0, d/m], \\ \min\{(d - c(j) \cdot s(j)/\gamma)/c(i), t(i)\}, \text{ if } c(j) \in (d/m, d \;/\; s(j)/\gamma), \\ 0, \text{ if } c(j) \in [d \;/\; s(j)/\gamma, c(i)). \end{cases}$$

To prove Proposition 13 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 14.
Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad m > s(j)/\gamma, \quad d/c(i) < s(j)/\gamma.$
Then

$$\inf_{c(j) \in (0, c(i))} \tau(i)(v(i); (s(j)/\gamma, c(j))) \;=\; 0.$$

To prove Proposition 14 we use Proposition 13.

Proposition 15.
Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad m > s(j)/\gamma, \quad d/c(i) = s(j)/\gamma.$
Then

$$\tau(i)(v(i); (s(j)/\gamma, c(j))) \;=\; = \begin{cases} \min\{d/c(i) \;\cdot\; (m \;-\; s(j)/\gamma)/m, t(i)\}, \; \textit{if}\; c(j) \in (0, d/m], \\ \min\{(d \;-\; c(j) \;\cdot\; s(j)/\gamma)/c(i), t(i)\}, \; \textit{if}\; c(j) \in (d/m, c(i)). \end{cases}$$

To prove Proposition 15 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 16.
Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad m > s(j)/\gamma, \quad d/c(i) = s(j)/\gamma.$
Then

$$\inf_{c(j) \in (0, c(i))} \tau(i)(v(i); (s(j)/\gamma, c(j))) \;=\; 0.$$

To prove Proposition 16 we use Proposition 15 and the definition of inf.

Proposition 17.
Let $i \in I$, $j \in I\setminus\{i\}$, $v(i) \in V(i)$, $\quad m > s(j)/\gamma, \quad d/c(i) > s(j)/\gamma, \quad d/c(i) < m.$
Then

$$\tau(i)(v(i); (s(j)/\gamma, c(j))) \;=\; = \begin{cases} \min\{d/c(i) \;\cdot\; (m \;-\; s(j)/\gamma)/m, t(i)\}, \; \textit{if}\; c(j) \in (0, d/m], \\ \min\{(d \;-\; c(j) \;\cdot\; s(j)/\gamma)/c(i), t(i)\}, \; \textit{if}\; c(j) \in (d/m, c(i)). \end{cases}$$

To prove Proposition 17 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 18.
Let $i \in I$, $j \in I\setminus\{i\}$, $v(i) \in V(i)$, $\quad m > s(j)/\gamma, \quad d/c(i) > s(j)/\gamma, \quad d/c(i) < m.$
Then

$$\inf_{c(j) \in (0, c(i))} \tau(i)(v(i); (s(j)/\gamma, c(j))) \;=\; \min\{d/c(i) \;-\; s(j)/\gamma, t(i)\}.$$

To prove Proposition 18 we use Propositions 8 and 17 and the definition of inf.

Proposition 19.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $m > s(j)/\gamma$, $d/c(i) > s(j)/\gamma$, $m \leqslant d/c(i)$. *Then*

$$\tau(i)(v(i); (s(j)/\gamma, c(j))) = \min\{m - s(j)/\gamma, t(i)\}$$

with all $c(j) \in (0, c(i))$.

To prove Proposition 19 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 20.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $m > s(j)/\gamma$, $d/c(i) > s(j)/\gamma$, $m \leqslant d/c(i)$. *Then*

$$\inf_{c(j) \in (0, c(i))} \tau(i)(v(i); (s(j)/\gamma, c(j))) = \min\{m - s(j)/\gamma, t(i)\}.$$

To prove Proposition 20 we use Proposition 19.

Proposition 21.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$. *The following holds*

$$\inf_{v(j) \in [0, s(j)/\gamma] \times (0, c(i))} \tau(i)(v(i); v(j)) = \begin{cases} 0, \ \textit{if } \mu(i)(v(i)) \leqslant s(j)/\gamma, \\ \min\{\mu(i)(v(i)) - s(j)/\gamma, t(i)\}, \ \textit{if } \mu(i)(v(i)) > s(j)/\gamma. \end{cases}$$

To prove Proposition 21 we use Propositions 10, 12, 14, 16, 18, 20 and the definition of the functions $\mu(i)$, $i \in I$.

Proposition 22.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $c(j) = c(i)$, $\mu(i)(v(i))/2 < s(j)/\gamma$. *Then*

$$\tau(i)(v(i); (t(j), c(j))) = \begin{cases} \min\{\mu(i)(v(i)) - t(j), t(i)\}, \ \textit{if } t(j) \in [0, \mu(i)(v(i))/2], \\ \min\{\mu(i)(v(i))/2, t(i)\}, \ \textit{if } t(j) \in (\mu(i)(v(i))/2, s(j)/\gamma). \end{cases}$$

To prove Proposition 22 we use the definitions of the functions $\tau(i)$ and $\mu(i)$, $i \in I$.

Proposition 23.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $\quad c(j) = c(i)$, $\quad \mu(i)(v(i))/2 \geqslant s(j)/\gamma$.

Then

$$\tau(i)(v(i);(t(j),c(j))) \;=\; \min\{\mu(i)(v(i)) \;-\; t(j), t(i)\}$$

with all $t(j) \in [0, s(j)/\gamma]$.

To prove Proposition 23 we use the definitions of the functions $\tau(i)$ and $\mu(i)$, $i \in I$.

Proposition 24.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

$$\inf_{(t(j),c(j)) \in [0,s(j)/\gamma] \times \{c(i)\}} \tau(i)(v(i);(t(j),c(j))) \;=$$

$$= \begin{cases} \min\{\mu(i)(v(i))/2, t(i)\}, \text{ if } \mu(i)(v(i))/2 \;<\; s(j)/\gamma, \\ \min\{\mu(i)(v(i)) \;-\; s(j)/\gamma, t(i)\}, \text{ if } \mu(i)(v(i))/2 \;\geqslant\; s(j)/\gamma. \end{cases}$$

To prove Proposition 24 we use Propositions 22 and 23 and the definition of the functions $\mu(i)$, $i \in I$.

Proposition 25.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$, $c(j) \in (c(i), +\infty)$.

Then

$$\tau(i)(v(i);(t(j),c(j))) \;=\; \min\{m, d/c(i), t(i)\}$$

with all $t(j) \in [0, s(j)/\gamma]$.

To prove Proposition 25 we use the definition of the functions $\tau(i)$, $i \in I$.

Proposition 26.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

$$\inf_{(t(j),c(j)) \in [0,s(j)/\gamma] \times (c(i),+\infty)} \tau(i)(v(i);(t(j),c(j))) \;=\; \min\{m, d/c(i), t(i)\}.$$

To prove Proposition 26 we use Proposition 25.

Proposition 27.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

$$\inf_{v(j) \in V(j)} \tau(i)(v(i); v(j)) = \\ = \begin{cases} 0, \text{ if } \mu(i)(v(i)) \leqslant s(j)/\gamma, \\ \min\{\mu(i)(v(i)) - s(j)/\gamma, t(i)\}, \text{ if } \mu(i)(v(i)) > s(j)/\gamma. \end{cases}$$

To prove Proposition 27 we use Propositions 8, 21, 24, 26 and the definition of the functions $\mu(i)$, $i \in I$.

Proposition 28.

Let $i \in I$, $j \in I \setminus \{i\}$, $v(i) \in V(i)$.

The following holds

1) $G(i)(v(i)) = s(i)$, *if* $c(i) \in (0, \lambda)$ *and* $t(i) = 0$,

2) $G(i)(v(i)) \leqslant s(i) - (\gamma - \lambda) \cdot t(i)$, *if* $c(i) \in (0, \lambda)$ *and* $t(i) \in (0, s(i)/\gamma]$,

3) $G(i)(v(i)) = s(i) - (\gamma - \lambda) \cdot t(i)$, *if* $c(i) = \lambda$,

4) $G(i)(v(i)) = s(i) - (\gamma - \lambda) \cdot t(i)$, *if* $c(i) \in (\lambda, +\infty)$ *and* $\mu(i)(v(i)) \leqslant s(j)/\gamma$,

5) $G(i)(v(i)) = s(i) - (\gamma - \lambda) \cdot t(i) + (c(i) - \lambda) \cdot \min\{\mu(i)(v(i)) - s(j)/\gamma, t(i)\}$, *if* $c(i) \in (\lambda, +\infty)$ *and* $\mu(i)(v(i)) > s(j)/\gamma$.

To prove Proposition 28 we use Propositions 3 and 27.

4. Conclusion

Expressions (given in Proposition 28) of players' guaranteed payoffs and also of upper evaluations of the guaranteed payoffs in the investigated game can be used to find sets of players' maxmin strategies in this game.

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