



## String models and Regge trajectories for baryons

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**Abstract.** The string with massive ends may be considered as the model  $q\bar{q}$  of a meson or as the quark-diquark model  $q\text{-}qq$  of a baryon. Rotational states (planar uniform rotations) of this string demonstrate quasilinear dependence between angular momentum and square of energy of a state. Such a behavior was used for describing light, strange, charmed, bottom mesons on Regge trajectories. In this paper we choose the quark-diquark string baryon model because of its stability and true Regge slope and use this model with two types of spin-orbit correction for describing  $N$ ,  $\Delta$ ,  $\Sigma$ ,  $\Lambda$  and  $\Lambda_c$  baryons on Regge trajectories.

**Keywords:** string models of baryons, rotational states, Regge trajectories

**MSC numbers:** 70B05, 70E55

## 1. Introduction

String models of mesons and baryons [1, 2, 3, 4, 5, 6] include Nambu-Goto string (relativistic string) connecting 2 or 3 massive points (quarks or antiquarks). This string simulates strong interaction between quarks at large distances, QCD confinement mechanism, it has constant energy density equal to the string tension  $\gamma$ .

Such a string with massive ends [2] may be regarded as the meson string model  $q\bar{q}$ , or the quark-diquark model  $q\text{-}qq$  [4] (on the classic level these models coincide). Other string models of baryons are: the linear configuration  $q\text{-}q\text{-}q$  [3, 7], the “three-string” model or Y configuration [6, 8], and the “triangle” model or  $\Delta$  configuration (a closed string with 3 massive points) [5, 6].

For all mentioned string hadron models one can use rotational states of these systems (classical planar uniform rotations) to describe quasilinear Regge trajectories for mesons and baryons [3, 4, 5, 9, 10, 11]. In the limit of large energies  $E$  for a rotational state the angular momentum  $J$  of this state behaves as

$$J \simeq \alpha' E^2 \quad (1)$$

for all mentioned models. For the meson model and baryon models  $q\text{-}qq$ ,  $q\text{-}q\text{-}q$  the slope  $\alpha'$  and the string tension  $\gamma$  are connected by Nambu relation [1]  $\alpha' = (2\pi\gamma)^{-1}$ . So, if we use these models with the same type of strings (the fundamental string), we can naturally describe baryonic and mesonic Regge trajectories with the same experimental slope  $\alpha' \simeq 0.9 \text{ GeV}^{-2}$ .

Rotational states of the string baryon model Y demonstrate the Regge asymptotics (1) with the slope [3, 4]  $\alpha' = 1/(3\pi\gamma)$ . To obtain the experimental value  $\alpha' \simeq 0.9 \text{ GeV}^{-2}$  we are to assume that the effective string tension  $\gamma_Y$  in this model differs from the fundamental string tension  $\gamma$  in models  $q\bar{q}$ ,  $q\text{-}qq$  and equals  $\gamma_Y = \frac{2}{3}\gamma$  [3, 9]. The string baryon model “triangle” or  $\Delta$  encounters the similar problem. For describing Regge trajectories with the so called triangle rotational states [5] we are to take another effective string tension  $\gamma_\Delta = \frac{3}{8}\gamma$  [3, 9].

To choose the most adequate string model of a baryon one should analyze the stability problem for rotational states of these models. Stability of classical rotational states with respect to small disturbances for the baryon models was studied in numerical experiments [6] and analytically [7, 8]. Rotational states for the linear model  $q\text{-}q\text{-}q$  and for the Y configuration appeared to be unstable. Analysis in Refs. [6, 7, 8] demonstrated that for both models small disturbance of a rotational state grow, this growth is exponential for the  $q\text{-}q\text{-}q$  model and it is linear for the Y model. These facts are very important for applications of these models in hadron spectroscopy.

Note that instability of classical rotations for a string configuration does not mean that the considered string model must be totally prohibited. All excited hadron states (objects of modelling) are resonances, they are unstable with respect to strong decays. So they have rather large width  $\Gamma$ . If classical rotations of a string configuration are unstable and this instability has a characteristic time scale  $t_{inst}$ , it gives the additional contribution  $\Gamma_{inst} \simeq 1/t_{inst}$  to width  $\Gamma$ . This effect takes place

for the linear model  $q-q-q$ , it restricts applicability of this string model, because the value  $\Gamma_{inst}$  predicted by this model for  $N$ ,  $\Delta$  and strange baryons in the mass (or energy) range 1 – 3 GeV essentially exceeds experimental data for  $\Gamma$  [7].

For the string model Y we see linear growth of small disturbances [8], this corresponds to zero contribution  $\Gamma_{inst} = 0$  in the increment of instability and in width  $\Gamma$  of baryons, described with the Y string model. So this rotational instability is not a weighty argument against application of the Y configuration. But this model has another drawback mentioned above, it predicts the slope  $\alpha' = (3\pi\gamma)^{-1}$  for Regge trajectories (1). This consideration makes us to choose the quark-diquark model  $q-qq$  for describing baryonic Regge trajectories. Rotations of this model are stable [6] and result in true Regge slope  $\alpha' = (2\pi\gamma)^{-1}$ .

In Sect. 2 of this paper we describe Regge trajectories for mesons and define optimal parameters of the string model, in Sect. 3 the same string with massive ends as the quark-diquark model is applied to describing baryons.

## 2. Regge trajectories for mesons

Rotational states of the string with masses  $m_1$ ,  $m_2$  at its ends are uniform rotations of a rectilinear string segment. They may be described as the following world surface  $X^\mu(\tau, \sigma)$  (a surface, swept by the string in Minkowski space  $R^{1,3}$ ) [6, 7, 11]:

$$X^\mu(\tau, \sigma) = \Omega^{-1} [\theta\tau e_0^\mu + \cos(\theta\sigma + \phi_1) \cdot (e_1^\mu \cos\theta\tau + e_2^\mu \sin\theta\tau)], \quad \sigma \in [0, \pi]. \quad (2)$$

Here  $\Omega$  is the angular velocity,  $e_0, e_1, e_2, e_3$  is the orthonormal tetrad in  $R^{1,3}$ . Values  $\theta$  and  $\phi_1$  are connected with the constant speeds  $v_j$  of the ends

$$v_1 = \cos\phi_1, \quad v_2 = -\cos(\pi\theta + \phi_1), \quad \frac{m_j\Omega}{\gamma} = \frac{1 - v_j^2}{v_j}.$$

Dynamical equations for the string with massive ends result from the action of this system [6, 11]. The expression (2) is the exact solution of these equations.

Rotational states (2) of the string with massive ends may be applied for describing excited mesons and baryons on Regge trajectories [3, 4, 9, 11], if we calculate energy  $E$  (or mass  $M = E$ ) and angular momentum  $J$  of a rotational state (2). Expressions for  $M$  and  $J$  have the following form [3, 9]:

$$M = \frac{\pi\gamma\theta}{\Omega} + \sum_{j=1}^2 \frac{m_j}{\sqrt{1 - v_j^2}} + \Delta E_{SL}, \quad (3)$$

$$J = L + S = \frac{1}{2\Omega} \left( \frac{\pi\gamma\theta}{\Omega} + \sum_{j=1}^2 \frac{m_j v_j^2}{\sqrt{1 - v_j^2}} \right) + \sum_{j=1}^n s_j. \quad (4)$$

Here  $s_j$  are spin projections of massive points,  $\Delta E_{SL}$  is the spin-orbit contribution to the energy [3, 9]

$$\Delta E_{SL} = \sum_{j=1}^2 \beta(v_j)(\Omega \cdot \mathbf{s}_j), \quad \beta(v_j) = 1 - (1 - v_j^2)^{1/2}. \quad (5)$$

This form of the spin-orbit contribution results from the assumption about pure chromoelectric field in the rotational center rest frame [3, 4]. The authors of Ref. [4] used the alternative expression

$$\beta(v_j) = 1 - (1 - v_j^2)^{-1/2}, \quad (6)$$

corresponding to the Thomas precession of the spins  $\mathbf{s}_j$ .

If the string tension  $\gamma$ , values  $m_j$  and  $s_j$  are fixed, we obtain an one-parameter set of rotational states (2). Values  $J$  and  $E^2$  for these states form the quasilinear Regge trajectory with asymptotic behavior (1) for large  $E$  and  $J$  [3] with the slope  $\alpha' = 1/(2\pi\gamma)$ .

We used the model of string with massive ends (considered as the model  $q\bar{q}$  of a meson) with spin-orbit correction (5) to describe experimental data for excited states of mesons in Ref. [11]. For this purpose we chose free parameters of the model: effective value of string tension  $\gamma$  and effective quark masses  $m_j$  for all flavors. We used the optimization procedure for choosing these effective values.

In this paper we apply to baryons the approach developed in Ref. [11], and also include both types of spin-orbit correction (5). The main principle of this choice is to describe the whole totality of experimental data on excited mesons and baryons [12].

At the first stage in Ref. [11] we describe main Regge trajectories for light unflavored mesons, strange mesons and choose effective values of tension  $\gamma$ , mass  $m_{ud}$  of the lightest quarks  $u$  and  $d$  (we suppose below that they are equal:  $m_u = m_d = m_{ud}$ ) and mass  $m_s$  of the strange quark  $s$ . These values are obtained in the following optimization procedure. We fix a set of  $n$  mesons with masses  $M_k$  and angular momenta  $J_k$ ,  $k = 1, \dots, n$ , with the definite quark composition. These mesons may lie on one or on a few Regge trajectories differing in quark spin  $S$  or isospin  $I$ . Such a set for light unflavored mesons is shown in Fig. 1, it includes 4 Regge trajectories.

For the best fitting between the model dependence  $J = J(M)$  (3), (4) and the experimental values  $M_k$ ,  $J_k$  from the table [12] for this set of mesons we use the least-squares method and minimize the sum of squared deviations with positive weights  $\rho_k$ :

$$F(m_1, m_2, \gamma) = \sum_{k=1}^n \rho_k \left[ J_k - J(M_k) \right]^2. \quad (7)$$

The weights  $\rho_k$  correspond to data or model errors, they are fixed below in the following manner:  $\rho_k = 1$  for reliable meson states from summary tables [12] with orbital momenta  $L \geq 1$ ;  $\rho_k = 0.2$  for unreliable states with high  $J$  omitted from summary tables [12], in particular, for  $\rho_5(2350)$  in Fig. 1, but  $\rho_k = 0.1$  for states of this type, that need confirmation ( $\omega_5$ ,  $a_6$ ,  $f_6$ );  $\rho_k = 0.2$  for states with  $L = 0$ . States with  $L = 0$ , in particular,  $\pi$ ,  $\rho(770)$ ,  $\eta$ ,  $\omega(782)$  in Fig. 1 should not be described by string models, because the string shape may correspond only to extended hadron states with high  $J$  [1, 2, 3, 4].

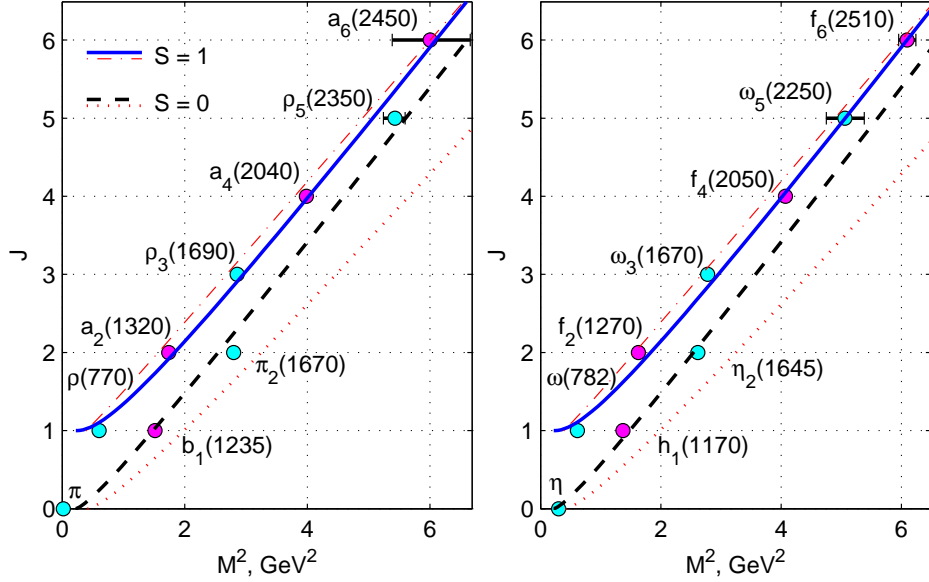


Figure 1: Regge trajectories with corrections (5) (heavy lines) and (6) (thin lines) (a) for isovector mesons  $\rho$ ,  $a$ ,  $\pi$ ; (b) for isoscalar mesons  $\omega$ ,  $f$ ,  $\eta$ .

To determine theoretical values of angular momenta  $J(M_k)$ , corresponding to masses  $M_k$  from the table [12], we are to invert numerically the function  $M(\Omega)$  (3) and substitute the function  $\Omega = \Omega(M)$  into Eq. (4):  $J(M_k) = J(\Omega(M_k))$ . We calculate the sum (7) for the mesons in Fig. 1 with both types of spin-orbit correction (5) and (6) for different effective values  $\gamma$  and  $m_{ud}$  (here  $m_1 = m_2 = m_{ud}$ ). The results of this calculation for 4 Regge trajectories from Fig. 1 are presented in Fig. 2 as level lines of the function (7) in the  $(m_{ud}, \gamma)$  plane.

We see that the sum (7) for the model (5) reaches its minimum  $F_{min} \simeq 0.18$ . The similar minimum  $F_{min} \simeq 0.94$  for the model (6) is 5 times larger. So this model is less successful in describing the mesons in Fig. 1 (thin lines).

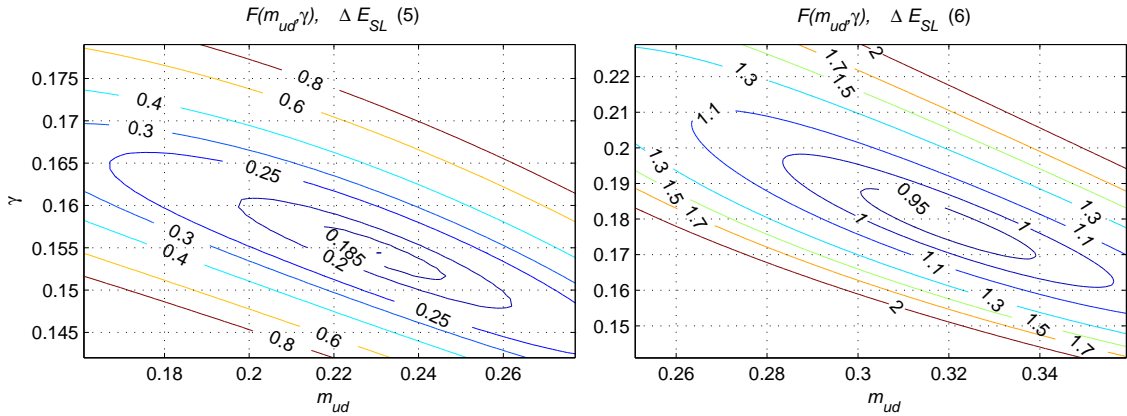


Figure 2: Level lines of the sum (7)  $F(m_{ud}, \gamma)$  for spin-orbit correction (5) (left) and (6) (right).

The optimal values of the model parameters  $\gamma$  and  $m_{ud}$  are presented below in Table 1. These values are fixed here with taking into account 8 leading Regge trajectories of mesons: to 4 mentioned above trajectories we add 4 sets of mesons with the strange quark  $s$ . They include 2 sets of  $K$  mesons with  $m_1 = m_{ud}$  and  $m_2 = m_s$ , in particular,  $K$ ,  $K_1(1270)$ ,  $K_2(1770)$  with summary quark spin  $S = 0$  and  $K^*(892)$ ,  $K_2^*(1430)$ ,  $K_3^*(1780)$ ,  $K_4^*(2045)$ ,  $K_5^*(2382)$  with  $S = 1$ . The sets  $\eta$ ,  $h_1(1380)$ ,  $\eta_2(1870)$  with  $S = 0$  and  $\phi(1020)$ ,  $f_2'(1525)$ ,  $\phi_3(1850)$  with  $S = 1$  are supposed to be  $s\bar{s}$  mesons:  $m_1 = m_2 = m_s$ . The weights  $\rho_k$  in Eq. (7) are determined as mentioned above, for example,  $\rho_k = 0.2$  for  $K$ ,  $K^*(892)$ ,  $\dots$   $h_1(1380)$ ,  $\rho_k = 0.1$  for  $K_5^*$  and  $\eta_2$ .

The sum (7) for these 8 Regge trajectories with 32 mesons depends on 3 parameters:  $F = F(m_{ud}, m_s, \gamma)$ . The minimal values  $F_{min} \simeq 0.301$  for the model (5) and  $F_{min} \simeq 1.267$  (4 times larger) for the case (6) are reached, if these 3 parameters take the values in Table 1. For the model (5) these values slightly differ from their analogs in Ref. [11]  $\gamma = 0.159$  GeV<sup>2</sup>,  $m_{ud} = 212.8$  MeV,  $m_s = 357.6$  MeV, but they appreciably differ from the effective parameters used in Ref. [3]  $\gamma = 0.175$  GeV<sup>2</sup>,  $m_{ud} = 130$  MeV,  $m_s = 270$  MeV. The values from Table 1 essentially diminish the sum  $F(m_{ud}, m_s, \gamma)$  in comparison with the parameters from Ref. [3].

Table 1: Effective values of parameters  $\gamma$  in GeV<sup>2</sup>, quark masses  $m_{ud}$ ,  $m_s$ ,  $m_c$ ,  $m_b$  in MeV for spin-orbit corrections (5) and (6).

Correction	$\gamma$	$m_{ud}$	$m_s$	$m_c$	$m_b$
(5)	0.154	231.5	369.0	1537.2	4818.0
(6)	0.1767	320.0	436.0	1532.2	4819.8

Description of 4 Regge trajectories of mesons with  $s$  quark is presented in Fig. 3. Here we use the same notations: heavy solid lines for the case (5)  $S = 1$ , dashed lines for  $S = 0$ , thin dash-dotted and dotted lines for the case (6). One can see, that the model with spin-orbit contribution (5) demonstrate better agreement.

If parameters  $\gamma$ ,  $m_{ud}$ ,  $m_s$  of the model are fixed in Table 1, we can determine the best value  $m_c$  of  $c$  quark for describing  $D$  mesons and charmonium states. They form 6 Regge trajectories:  $D$ ,  $D_1(2420)$ ,  $D^*$ ,  $D_2^*(2460)$  are charmed analogs of  $K$  and  $K^*$  mesons;  $D_s$ ,  $D_{s1}(2536)$ ,  $D_s^*$ ,  $D_{s1}^*(2573)$  are their partners  $c\bar{s}$  or  $s\bar{c}$ ;  $\eta_c$ ,  $h_c(1P)$ ,  $J/\psi$ ,  $\chi_{c2}$  are  $c\bar{c}$  states. As stated above we take  $\rho_k = 0.2$  for mesons with  $L = 0$  and  $\rho_k = 1$  for  $L = 1$ . Under these circumstances we minimize the function (7)  $F = F(m_c)$  of one argument and obtain the optimal values  $m_c$  in Table 1 for both considered models with spin-orbit corrections (5) and (6).

Six Regge trajectories for charmed mesons are presented in Fig. 3. Note that for them we used only one fitting parameter  $m_c$ , but all trajectories are described rather well in both models with Eqs. (5) and (6).

The similar approach to bottom mesons  $B$ ,  $B^*$ ,  $B_s$ ,  $B_s^*$ ,  $\Upsilon$ ,  $\chi_{b2}(1P)$  with sub-

stitution  $c$  for  $b$  quark results in the optimal values  $m_b$  for bottom quark in Table 1 and corresponding Regge trajectories for bottom mesons in Fig. 3. Here we use the notations of Fig. 1, in particular, thick lines correspond to spin-orbit correction (5). This model has advantage in comparison with the case (6) (thin lines) for mesons with strange quark  $s$ . For charmed and bottom mesons this advantage becomes inessential. Both models are successful in describing charmonium states, but they work worse for bottomonium.

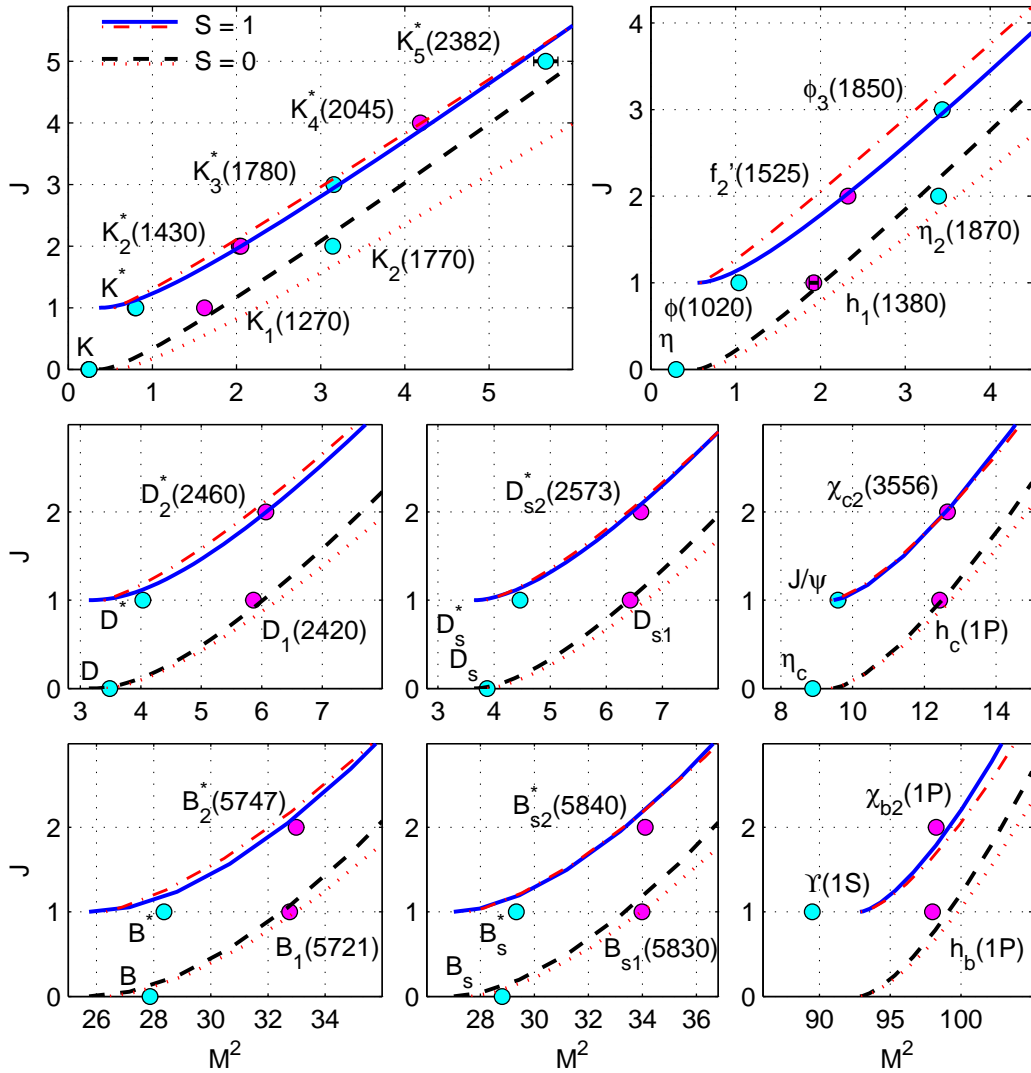


Figure 3: Regge trajectories for strange, charmed, bottom mesons with model parameters  $\gamma$ ,  $m_q$  from Table 1.



### 3. Regge trajectories for baryons

The same rotational states (2) of the string with massive ends and formulas (3), (4) for values  $M$  and  $J$  may be applied to Regge trajectories for baryons. We mentioned above that for describing baryonic Regge trajectories we are to choose the quark-diquark string model. Only this model predicts rotational stability and natural Regge slope  $\alpha' = (2\pi\gamma)^{-1}$  corresponding to equal experimental Regge slopes  $\alpha'$  both for mesons and baryons. Therefore, to describe baryons we can use the same approach with the same spin-orbit corrections (5) or (6) and with optimal model parameters from Table 1 for the tension  $\gamma$  and masses  $m_j$  of a single quark in quark-diquark systems.

We represent the model of a baryon as a string with two massive ends with mass  $m_1$  of the single quark and effective diquark mass  $m_2$ . The value  $m_1$  depends on a baryon under consideration and equals  $m_{ud}$ ,  $m_s$ ,  $m_c$  or  $m_b$  from Table 1.

There are several approaches to determine the mass  $m_2$ . In the simplest case, one can assume that diquarks are weakly bound systems, so a diquark mass is close to sum of two constituent quark masses [3], in particular, for  $N$  and  $\Delta$  baryons  $m_d \simeq 2m_{ud}$ . Another approach is widely used in string and potential quark-diquark models of baryons [4, 13, 14] and supposes different diquark masses for scalar diquarks with total spin  $S_d = 0$  and for vector diquarks with  $S_d = 1$ . Essential difference of these masses corresponds to strong coupling between two quarks in a diquark. However this mechanism remains vague, the diquark masses in the mentioned models are used as fitting parameters. In particular, masses  $m_d^0$  of the scalar  $[u, d]$  diquark in  $N$  baryons and  $m_d^1$  for the vector  $\{u, d\}$  diquark in  $\Delta$  baryons in the potential model [14] are correspondingly 710 and 909 MeV. The similar masses in the string model [4] are  $m_d^0 = 220$  and  $m_d^1 = 550$  MeV. This difference requires special explanation.

The last approach may be applied to our string model with two forms (5) and (6) of spin-orbit correction. For this purpose we use two sets of baryonic Regge trajectories with scalar and vector diquarks correspondingly. For scalar diquarks we choose the main Regge trajectory for  $N$  baryons  $N$ ,  $N(1520)$ ,  $N(1680)\dots$  and the corresponding trajectory for  $\Lambda$  baryons:  $\Lambda$ ,  $\Lambda(1520)$ ,  $\Lambda(1820)\dots$ . We use data for mesons from Table 1 and fix weights  $\rho_k$  similarly to the mesons, in particular, we suppose  $\rho_k = 0.2$  for  $N$ ,  $\Lambda$ ,  $N(2700)$ ,  $\Lambda(2585)$ ,  $\rho_k = 1$  for reliable baryons.

If we fix tension  $\gamma$ , the mass  $m_1 = m_{ud}$  or  $m_s$  of a single quark and vary the free parameter  $m_d = m_d^0$ , we calculate the dependence of the sum (7) on  $m_d$ . This dependence is shown in Fig. 4 (l.h.s.) for the models (5) (solid lines) and (6) (dash-dotted lines). The graphs in r.h.s. present the similar function  $F(m_d^1)$  for the vector diquark with  $S_d = 1$ . They are calculated on the base of trajectories with  $\Delta$  baryons  $\Delta(1232)$ ,  $\Delta(1930)$ ,  $\Delta(1950)\dots$  and  $\Sigma$  baryons  $\Sigma(1385)$ ,  $\Sigma(1775)$ ,  $\Sigma(2030)$  with the same approach for weights  $\rho_k$ .

Graphs  $F(m_d)$  in Fig. 4 let us determine their points of minimum, that is the optimal values  $m_d^0$  and  $m_d^1$  for describing the mentioned Regge trajectories. These optimal values for the scalar and vector diquark masses MeV are presented in



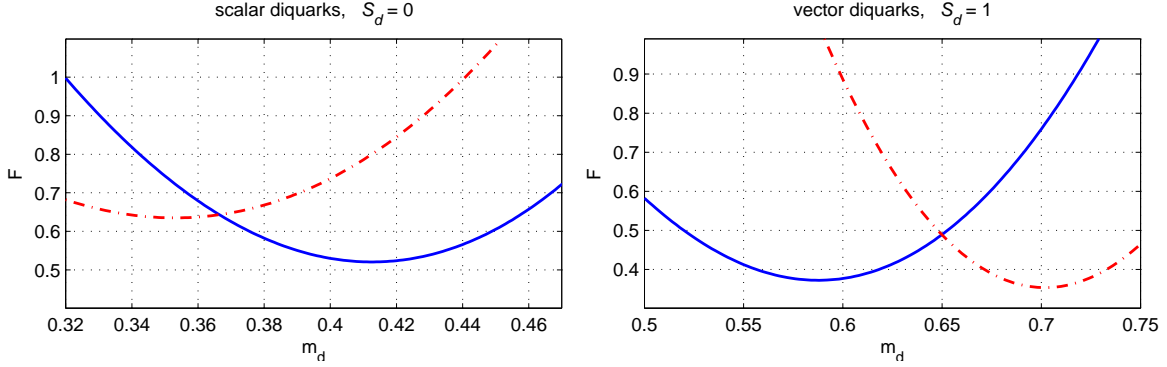


Figure 4: Graphs  $F(m_d)$  for baryons with scalar diquarks (left) and vector diquarks (right) in models (5) (solid lines) and (6) (dash-dotted lines).

Table 2 for both considered models.

Table 2: Diquark masses  $m_d$  for models (5) and (6).

Correction	$m_d^0$ for $S_d = 0$	$m_d^1$ for $S_d = 1$
(5)	412.6	588
(6)	352.9	702

One can see that in the model with spin-orbit correction (6) optimal masses of scalar and vector diquarks are essentially different:  $m_d^1 \simeq 2m_d^0$ . This difference corresponds to so the similar relation between  $m_d^1$  and  $m_d^0$  in Ref. [4], where the same correction (6) in the quark-diquark model was used.

This difference is not so large in the model with spin-orbit correction (5), the value  $m_d^0$  in Table 2 is close to  $2m_{ud} = 463$  MeV. So in this model we can use not only optimal parameters from Table 2, but also  $m_d = 2m_{ud}$  (see Fig. 5).

Regge trajectories for the mentioned baryons are shown in Fig. 5. The model parameters for both models (5) and (6) are taken from Tables 1, 2, but for the model (5) we present two variants of the diquark mass: predictions with  $m_d^0 = 412.6$  and  $m_d^1 = 588$  MeV from Table 2 are shown as blue thick solid lines but the similar graphs with  $m_d = 2m_{ud} = 463$  MeV are described with dotted lines. For the model (6) we present only the case with  $m_d^0$  and  $m_d^1$  from Table 2, these curves are shown with red dash-dotted lines (similarly to Fig. 3). The choice  $m_d = 2m_{ud}$  for this model works much worse because large difference between the optimal values  $m_d^0$  and  $m_d^1$  in Table 2.

Distribution of quark spins in Fig. 5 is obvious for  $N$ ,  $\Lambda$ ,  $\Lambda_c$  baryons with  $s_1 = 1/2$ ,  $s_2 = S_d = 0$  and for baryons with  $S = 3/2$  ( $s_1 = -1/2$ ,  $s_2 = 1$ ). Baryons  $\Delta(1700)$ ,  $\Delta(1905)$ ... and  $\Sigma$ ,  $\Sigma(1670)$ ,  $\Sigma(1915)$  are to be interpreted as states with  $S = 1/2$  and the most natural spin distribution for them is  $s_1 = -1/2$ ,  $s_2 = 1$ .

We see in Fig. 5 that for main (parent) Regge trajectories for  $N$ ,  $\Lambda$  baryons with

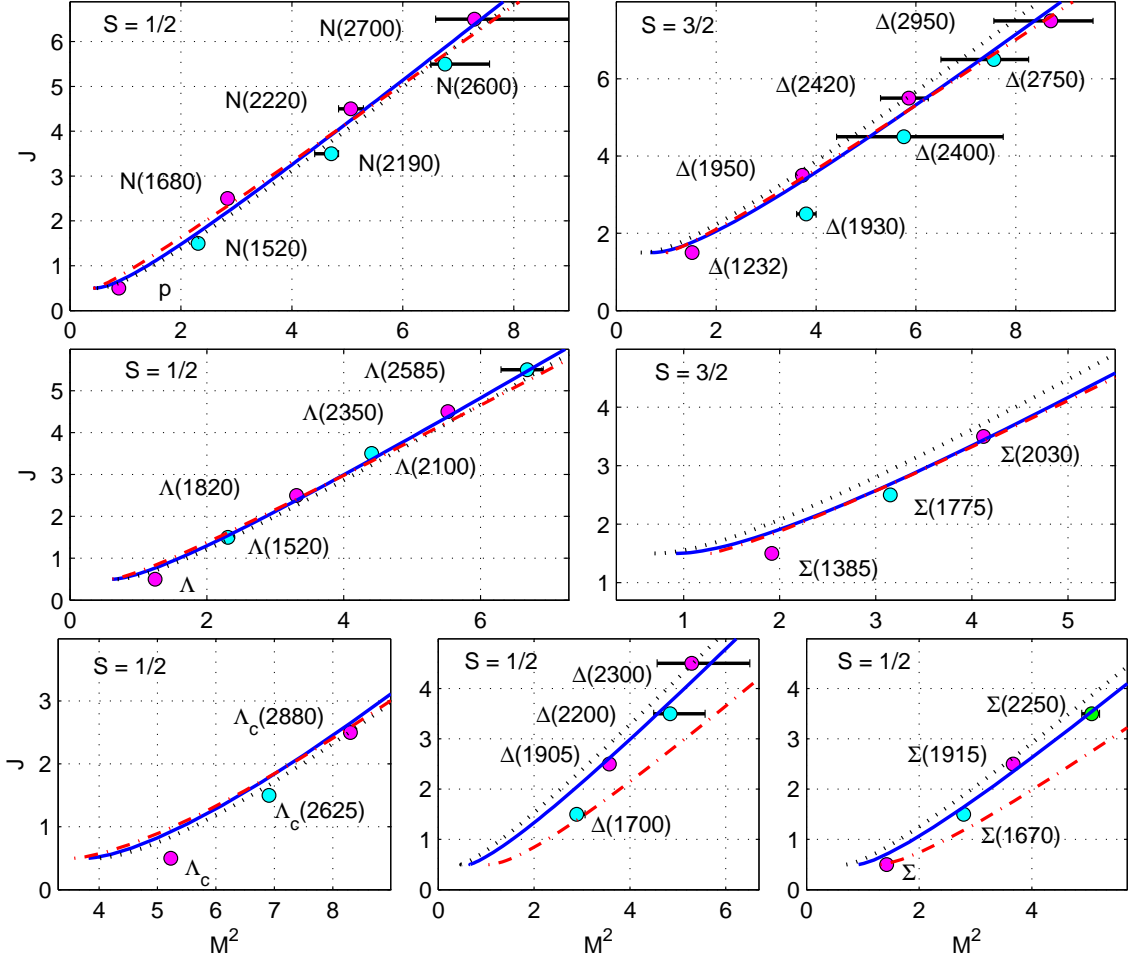


Figure 5: Regge trajectories for baryons in models (5) (solid lines for  $m_d$  from Table 2 and dotted lines for  $m_d = 2m_{ud} = 463$  MeV) and (6) (dash-dotted lines).

$S = 1/2$  and for  $\Delta$  and  $\Sigma$  baryons with  $S = 3/2$  predictions of both models (5) and (6) with  $m_d^j$  from Table 2 are very close. For  $\Delta$  and  $\Sigma$  baryons with  $S = 3/2$  these curves practically coincide.

The similar coincidence takes place for charmed  $\Lambda_c$  baryons (only these charmed baryons form an appreciable Regge trajectory). The trajectory for  $\Lambda_c$  baryons is used here as a test for these models, all their parameters  $\gamma$ ,  $m_c$ ,  $m_d^0$  were determined previously.

But for  $\Sigma$  baryons  $\Sigma$ ,  $\Sigma(1670)$ ,  $\Sigma(1915)$  and  $\Delta$  baryons  $\Delta(1700)$ ,  $\Delta(1905)$ ... the models (5) and (6) predict different Regge trajectories, if we interpret these hadrons as states with  $s_1 = -1/2$ ,  $s_2 = S_d = 1$ . In this approach only the model with correction (5) works successfully, in the case (6) the mass correction  $\Delta E_{SL}$  appears to be positive and too large. So we can describe these Regge trajectories in the model (6) only if we suppose that diquarks in these hadrons are scalar ones. In this case trajectories for  $\Delta$  and  $\Sigma$  baryons with  $S = 1/2$  will be copies of trajectories for  $N$  and  $\Lambda$  baryons.

Dotted lines for all baryons in Fig. 5 show that the model with correction (5) admits the diquark mass  $m_d = 2m_{ud}$ . This assumption works rather good for  $N$ ,  $\Lambda$  and  $\Lambda_c$  baryons and it works worse for  $\Delta$  and  $\Sigma$  baryons. Note that the model with correction (6) is incompatible with the assumption  $m_d = 2m_{ud}$ .

## 4. Conclusion

Rotational states (2) of the string with massive ends with their Regge behavior (3), (4) are applied to describing Regge trajectories for mesons and baryons. For baryons this model is considered as the quark-diquark model  $q$ - $qq$ . It has some advantages in comparison with other baryon models. In particular, for the linear configuration  $q$ - $q$ - $q$  [3, 7] and for the Y model [6, 8] rotational states are unstable, small disturbances grow linearly for Y and exponentially for the linear model. This results in too large additional width  $\Gamma$  of excited baryons in the linear model [7].

For the Y string baryon model we have no additional width, but this model predicts the slope  $\alpha' = (3\pi\gamma)^{-1}$  for Regge trajectories, that differs from  $\alpha' = (2\pi\gamma)^{-1}$  for the string with massive ends [3, 4]. So for describing both mesons and baryons with almost equal experimental value of  $\alpha'$  we have to use the string with massive ends as the meson model  $q$ - $\bar{q}$  and as the quark-diquark baryon model  $q$ - $qq$ .

These models with spin-orbit correction in two forms (5) and (6) can describe main Regge trajectories for light unflavored mesons, for  $K$ ,  $D$ ,  $D_s$ ,  $B$ ,  $B_s$  mesons, charmonium and bottomonium states, and also for  $N$ ,  $\Delta$ ,  $\Sigma$ ,  $\Lambda$  and  $\Lambda_c$  baryons. In this approach we use the optimization procedure with choosing the effective string tension  $\gamma$  and effective masses of quarks and diquarks  $m_j$  for all flavors (see Tables 1, 2).

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