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Neutron-deuteron inelastic scattering: numerical modelling and asymptotic conditions

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Abstract. Inelastic neutron-deuteron scattering is studied on the basis of configuration-space Faddeev equations. Calculated are neutron-deuteron breakup amplitudes using AV14 nucleon-nucleon potential at incident neutron energy of 14.1 MeV. The results are presented for the differential cross sections under quasi free scattering (QFS) and space-star (SST) configurations, and compared with those of the previous calculations and experimental data. The choice of the cutoff radius R_{cutoff} for asymptotic conditions is discussed.

Keywords: Few-body systems, Faddeev equation, Inelastic scattering

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1. Introduction

The last two decades brought tremendous progress in both theoretical and experimental study of nucleon-deuteron scattering. More accurate experimental data were compared with calculations in the framework of the Faddeev equations with high-precision nucleon-nucleon potentials also including model three-nucleon forces in Ref. [1, 2]. It can be noted that there are several important cases where the 3N calculations have failed to explain the data [3]. Among the most important discrepancies are the $\mathrm{A_y}$ puzzle in nucleon-deuteron (Nd) elastic scattering [4], the star configuration in the Nd breakup reaction [5] and the quasi-free scattering (QFS) cross section [1]. Thus the situation with theoretical predictions for the 3N observables remains still far from completely resolved. This motivates us to continue searching for new computational procedures and input potentials, which may allow to overcome the mentioned disagreements.

In this work, we present a mathematically rigorous approach for the nd breakup scattering problem. It includes a generalization of the formalism for three-body scattering in configuration space initiated by Merkuriev et al. [6], to calculate nd breakup amplitudes with modern nucleon-nucleon potentials.

We focus on the problem of choice of model space for numerical solution of the inelastic scattering problem. In particular, the appropriate choice of cutoff radius R_{cutoff} in asymptotic conditions is important for the scattering observable calculations [7, 8]. In Ref. [7], the such analyses for the Faddeev equation in configuration space has been fulfilled for model three-body system. In Ref. [8] the consideration was restricted by using s-wave potentials.

In present work we perform numerical analysis of the problem involving the nd breakup observable calculations with realistic AV14 potential. We implements the extrapolation procedure for calculation of amplitudes with respect to the cutoff radius proposed in [7].

2. Model description

We study inelastic neutron-deuteron scattering on the basis of the configuration-space Faddeev equations. The MGL basis proposed in [6] is applied to perform partial wave analysis (also, see [9]). An illustration for definition of the quantum numbers for MGL basis is given in Fig. 1.

The formalism of the Faddeev equations is given in the Appendix I. Here we formulate the asymptotic boundary conditions for the Faddeev components $\Phi_\alpha^{\lambda_0 s_0 M_0}(x,y)$ of the wave function. The asymptotic conditions for nd breakup scattering has the following form [10]:

$$\begin{aligned}\Phi_\alpha^{\lambda_0 s_0 M_0}(x,y) \sim \Big\{ \Big[\delta_{\lambda\lambda_0}\delta_{ss_0}\delta_{\sigma 1}\delta_{j1}\hat{j}_\lambda(qy) + \Big(-\hat{y}_\lambda(qy) + \imath\hat{j}_\lambda(qy)\Big) a^{M_0}_{\lambda s\lambda_0 s_0}\Big]\psi_l(x) \\ +O(y^{-1})\Big\} + A^{M_0}_{\alpha,\lambda_0 s_0}(\theta)\frac{e^{\imath KX}}{\sqrt{X}} + O(X^{-3/2}),\end{aligned} \tag{1}$$

$$X^2 = x^2 + y^2, \quad K^2 = \frac{mE}{\hbar^2},$$

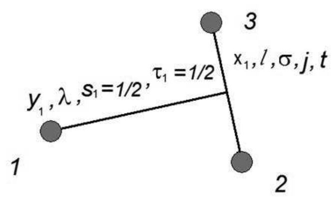

Figure 1: Definition of the quantum numbers for partial wave analysis of the Faddeev equations in the MGL basis. $\mathbf{M}$ is the total three particle momentum $\mathbf{M} = \boldsymbol{\lambda} + \mathbf{s}$, where $\mathbf{s} = \mathbf{1/2} + \mathbf{j}$.

where x is finite, $y \to \infty$. ψ_l is $l - th$ component of deuteron wave function (l =0,2), and $\hat{j}$ and $\hat{y}$ are the regularized spherical Bessel functions.

The matrix of partial elastic amplitudes a has the structure

$$a^{M_0}_{\lambda s \lambda_0 s_0} = \frac{\eta \exp(2\imath\delta) - 1}{2\imath}, \tag{2}$$

where η and δ are the inelasticity and scattering phase. In Eq. (1) the amplitudes $A^{M_0}_{\alpha,\lambda_0 s_0}$ are related with total breakup amplitudes by the sum

$$\mathcal{A}^{M_0}_{\alpha,\lambda_0 s_0}(\theta) = A^{M_0}_{\alpha,\lambda_0 s_0}(\theta) + \int_{-1}^{1} du \sum_{\beta} g_{\alpha\beta}(\theta, u) A^{M_0}_{\beta,\lambda_0 s_0}(\theta'). \tag{3}$$

The nd observables are calculated with the charge independent AV14 nucleon-nucleon potential. For the calculation of the breakup amplitudes we take into account the total angular momentum of a pair nucleons $j \leq 3$, and the total three-body angular momentum M up to 13/2. The formalism used for calculations of the nd observables is given in the Appendix II.

3. Results

A new computational method including spline-decomposition in angular variable θ and the Numerov method for the hyperradius ρ [9, 11] is implemented for the solving nucleon-deuteron breakup scattering problem. The finite-difference approximation of Eq. (15) is performed over the greed of the angular and radial variables. One needs to choice the cutoff radius R_{cutoff} which defines the asymptotic domain when the representation of Eq. (1) becomes valid. According Ref. [7], the appropriate radius has to be about 100 fm.

For elastic observables we obtain a good agreement with the predictions of the Bochum [1, 12] and Grenoble [13] groups using this value for R_{cutoff}. Our results agrees also with experimental data (see also, our work [9]).

Inelastic observables are calculated with set of the R_{cutoff} values of 80 fm, 100 fm, and 120 fm. In Fig. 2 and 3 we show the angular distributions of the nd differential cross sections calculated for the QFS and SST configurations. The

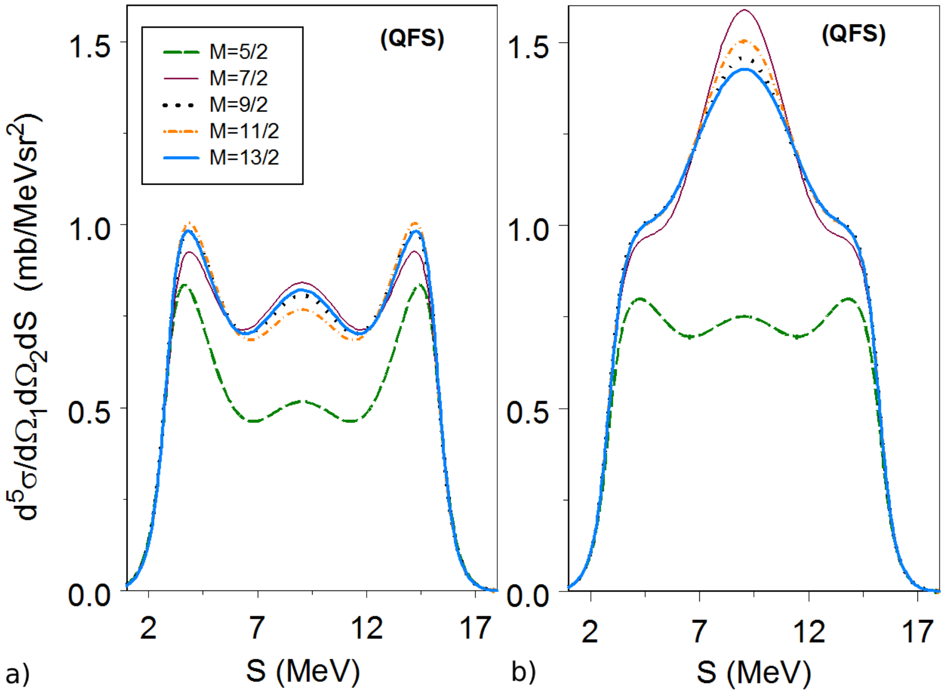

Figure 2: The nd differential cross sections as a functions of arc length S. for the quasi-free scattering configuration (QFS) of the final nucleons at E=14.1 MeV (θ_1=39°, θ_2=39°, ϕ=180°). a) The results of calculation obtained for R_{cutoff}=120 fm. b) The three point extrapolation for R_{cutoff} at 80 fm, 100 fm, and 120 fm. The solid line corresponds to calculation maximal partial wave taken into account with M=13/2. Other notations are given in the legend.

results are compared with the experimental data and the prediction of the Bochum group. Convergence of numerical results with respect to the maximum value of the three-body angular momentum M was tested for the QFS configuration. Corresponding results are presented in Fig. 2. The value of M up to 11/2 provides sufficient accuracy of the calculation.

Application of the three point extrapolation with respect to the cutoff radius [7] is absolutely needed for both configurations to obtain qualitative agreement with the experimental data. The extrapolation has the following form:

$$A_{calc} \to A(\theta) + \frac{1}{k_0\rho}B(\theta) + \frac{1}{(k_0\rho)^2}C(\theta) + \dots,$$

where $A(\theta)$ is the true breakup amplitude. The extrapolation allows us to obtain solution for limit $\rho \to \infty$ where the asymptotic conditions (1) is exactly satisfied. For calculation, the use of the extrapolation reduces the computational difficulties arising for large values of the cutoff radius.

In Fig. 2 we show the difference between the QFS configurations results obtained for the R_{cutoff}=120 fm calculation and the three point extrapolation for R_{cutoff} at

80 fm, 100 fm, and 120 fm. Existence of difference means that the R_{cutoff} values about 100 fm do not satisfy the asymptotic domain for Eq. (1) for inelastic scattering. To obtain reliable accuracy for calculation of the differential cross sections under the QFS configuration, R_{cutoff} has to be considerably greater than 120 fm.

For the SST configuration we obtain slightly different shape of the cross section curve, if compared to the prediction of the Bochum group (see Fig. 3). However, in the vicinity of the exact space-star position the agreement between our results and the experimental data is very impressive. We have to note the weak sensitivity on R_{cutoff} of the calculation for the SST configuration. The extrapolated values are close to ones of the R_{cutoff}=120 fm calculation.

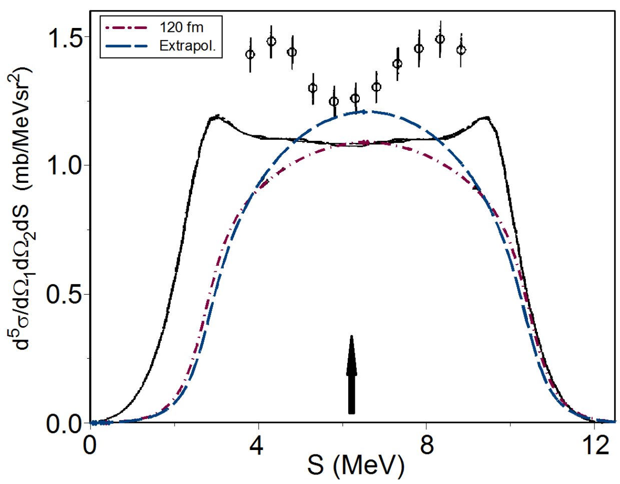

Figure 3: The nd differential cross sections as a functions of arc length S. The space-star configuration (SST) of the final nucleons at E=14.1 MeV (θ_1=51.01°, θ_2=51.01°, ϕ=120°) for R_{cutoff} at 120 fm (dot-dashed line). The dashed line corresponds to three point extrapolation. The Bochum group result [1] is shown by solid line. The experimental data (circles) for E_{lab}=13.0 MeV are taken from [12]. The exact space-star situation is marked by an arrow.

4. Conclusions

A new numerical method for solving the Faddeev equations in configuration space was implemented for calculation of the nd breakup observables at E_{lab} = 14.1 MeV. The numerical results are converged for M_{max} ≤11/2 with good accuracy. We found that, the value of cutoff radius much greater than 120 fm is required what the asymptotic conditions for the breakup scattering were accomplished. The calculation for $R_{cutoff} \to \infty$ can be fulfilled by the extrapolation procedure for amplitudes with respect to the cutoff radius.

5. Appendix I

The starting point for studying interactions between nucleons in three-body systems is the solution of the Schrödinger equation $H\Psi = E\Psi$ for nuclear Hamiltonian

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{3}\nabla_i^2 + \sum_{j<k} V_{jk} \quad \Big(+ \sum_{j<k<l} V_{jkl}\Big), \tag{4}$$

where V_{jk} and V_{jkl} are the two- and three-nuclear potentials, respectively. In this study we neglected three-nucleon forces V_{jkl}.

Writing the total wave function as

$$\Psi = \Phi_1 + \Phi_2 + \Phi_3 = (1 + P^+ + P^-)\Phi_1, \tag{5}$$

the Schrödinger equation for three identical particles can be reduced into a single Faddeev equation, which in Jacobi's vectors $\mathbf{x_1}, \mathbf{y_1}$ has the form

$$\Big[-\frac{\hbar^2}{m}\Big(\Delta_{\mathbf{x}_1} + \Delta_{\mathbf{y}_1}\Big) + V(\mathbf{x}_1) - E\Big]\Phi(\mathbf{x}_1, \mathbf{y}_1) = -V(\mathbf{x}_1)(P^+ + P^-)\Phi(\mathbf{x}_1, \mathbf{y}_1), \tag{6}$$

where the operators $P^{\pm}$ are the cyclic permutation operators for the three particles which interchange any pair of nucleons ($P^+ : 123 \to 231, P^- : 123 \to 312$), and $\hbar^2/m$=41.47 MeV·fm^2. As independent coordinates, we take the Jacobi vectors $\mathbf{x}_\alpha, \mathbf{y}_\alpha$. For the pair α=1, they are related to particle coordinates by the formulas:

$$\mathbf{x}_1 = \mathbf{r}_2 - \mathbf{r}_3, \qquad \mathbf{y}_1 = \frac{\mathbf{r}_2 + \mathbf{r}_3}{2} - \mathbf{r}_1, \tag{7}$$

for α=2,3 one has to make cyclic permutations of the indexes in Eq. (7). The Jacobi vectors with different α's are linearly related by the orthogonal transformation

$$\begin{pmatrix} \mathbf{x}_\alpha \\ \mathbf{y}_\alpha \end{pmatrix} = \begin{pmatrix} C_{\alpha\beta} & S_{\alpha\beta} \\ -S_{\alpha\beta} & C_{\alpha\beta} \end{pmatrix} \begin{pmatrix} \mathbf{x}_\beta \\ \mathbf{y}_\beta \end{pmatrix}, \quad C^2_{\alpha\beta} + S^2_{\alpha\beta} = 1, \tag{8}$$

where

$$C_{\alpha\beta} = -\sqrt{\tfrac{m_\alpha m_\beta}{(M-m_\alpha)(M-m_\beta)}}, \quad S_{\alpha\beta} = (-)^{\beta-\alpha}\mathrm{sgn}(\beta-\alpha)\sqrt{1 - C^2_{\alpha\beta}}, \quad M = \textstyle\sum_{\alpha=1}^{3} m_\alpha.$$

To perform numerical calculations for arbitrary nuclear potential, we use MGL approach [6]. To make the angular analysis of Eq. (6), we use a basis proposed in [6]. This basis is intermediate between LS and Jj coupling schemes.

Let: $\boldsymbol{\sigma}$, $\mathbf{l}$ and $\mathbf{J} = \boldsymbol{\sigma} + \mathbf{l}$ be the spin, orbital and total angular momenta of the pair 23. $\mathbf{s} = \mathbf{1/2} + \mathbf{J}$ be the total "spin" of the system 123 considering the pair 23 as a particle with "spin" $\mathbf{J}$. $\boldsymbol{\lambda}$ be the orbital momentum conjugate to $\mathbf{y}$, that is of the relative motion of particle 1 respective to the c.m of pair 23. $\mathbf{M} = \boldsymbol{\lambda} + \mathbf{s}$ be the total angular momentum with its z-projection M_z. To this we have to add the isospin part. If the isospin of the pair 23 is $\mathbf{t}$, then the total isospin is $\mathbf{T} = \mathbf{1/2} + \mathbf{t}$

with its projection T_z. Since in the nd case $T = 1/2$ it need not be shown explicitly. The set of quantum numbers $\{\lambda s l \sigma J\} \equiv \alpha$ defines a state in this basis.

Correspondingly in this basis the spin-angular-isospin eigenfunctions have the form

$$Z_\alpha(\hat{\mathbf{x}}, \hat{\mathbf{y}}) =< \hat{\mathbf{x}}, \hat{\mathbf{y}}|\alpha >= \left[Y^\lambda(\hat{\mathbf{y}}) \otimes [\chi^{1/2} \otimes \mathcal{Y}^J_{l\sigma}(\hat{\mathbf{x}})]^s \right]^{M,M_z} \eta^{T,T_z}_{1/2,t}. \tag{9}$$

For *nd* scattering the Faddeev equations for partial components can be written in the following form (here we omit the index 1):

$$\begin{gathered} \left[E + \tfrac{\hbar^2}{m}(\partial_x^2 + \partial_y^2) - v_\alpha^{\lambda l}(x, y) \right] \Phi_\alpha^{\lambda_0, s_0, M_0}(x, y) = \\ \sum_\beta v_{\alpha\beta}(x) \left[\Phi_\beta^{\lambda_0, s_0, M_0}(x, y) + \int_{-1}^{1} du \sum_\gamma g_{\beta\gamma}(y/x, u) \Phi_\gamma^{\lambda_0, s_0, M_0}(x', y') \right]. \end{gathered} \tag{10}$$

The geometrical function $g_{\beta\gamma}(x, y, u)$ is the representative of the permutation operator $P^+ + P^-$ in MGL basis [6]:

$$\begin{aligned} & g_{\alpha'\alpha}(y/x, u) = g_{\alpha'\alpha}(\theta, u) = g_{\alpha'\alpha}(\theta, \theta') \\ & = (-1)^{\lambda+\lambda'+J+J'}[(2J+1)(2J'+1)(2s+1)(2s'+1)]^{1/2} \sum_{LS}(2S+1)(2L+1) \\ & \times \left\{ \begin{array}{ccc} l & \sigma & J \\ 1/2 & s & S \end{array} \right\} \left\{ \begin{array}{ccc} l' & \sigma' & J' \\ 1/2 & s' & S \end{array} \right\} \left\{ \begin{array}{ccc} \lambda & l & L \\ S & M & s \end{array} \right\} \left\{ \begin{array}{ccc} \lambda' & l' & L \\ S & M & s' \end{array} \right\} \\ & \times < \chi^S_{1/2\sigma'} \eta^T_{1/2,t'} | P^+ | \chi^S_{1/2\sigma} \eta^T_{1/2,t} > h^L_{\lambda' l' \lambda l}(y/x, u). \end{aligned} \tag{11}$$

Function h is the representative of the permutation operator $P^+ + P^-$ in the $\boldsymbol{\lambda} + \mathbf{l} = \mathbf{L}$ basis:

$$\begin{aligned} & h^L_{\lambda' l' \lambda l}(y/x, u) = h^L_{\lambda' l' \lambda l}(\theta, u) = h^L_{\lambda' l' \lambda l}(\theta, \theta') \\ & = \frac{xy}{x'y'}(-1)^{l+L} \frac{(2\lambda+1)(2l+1)}{2^{\lambda+l}} [(2\lambda)!(2l)!(2\lambda'+1)(2l'+1)]^{1/2} \\ & \times \sum_{k=0}(-1)^k(2k+1)P_k(u) \sum_{\lambda_1+\lambda_2=\lambda,\, l_1+l_2=l} \frac{y^{\lambda_1+l_1} x^{\lambda_2+l_2}}{y'^{\lambda} x'^{l}}(-1)^{l_2} \\ & \times \frac{(\sqrt{3})^{\lambda_2+l_1}}{[(2\lambda_1)!(2\lambda_2)!(2l_1)!(2l_2)!]^{1/2}} \sum_{\lambda'' l''}(2\lambda''+1)(2l''+1) \\ & \times \left(\begin{array}{ccc} \lambda_1 & l_1 & \lambda'' \\ 0 & 0 & 0 \end{array} \right) \left(\begin{array}{ccc} \lambda_2 & l_2 & l'' \\ 0 & 0 & 0 \end{array} \right) \left(\begin{array}{ccc} k & \lambda'' & \lambda' \\ 0 & 0 & 0 \end{array} \right) \left(\begin{array}{ccc} k & l'' & l' \\ 0 & 0 & 0 \end{array} \right) \\ & \times \left\{ \begin{array}{ccc} l' & \lambda' & L \\ \lambda'' & l'' & k \end{array} \right\} \left\{ \begin{array}{ccc} \lambda_1 & \lambda_2 & \lambda \\ l_1 & l_2 & l \\ \lambda'' & l'' & L \end{array} \right\}. \end{aligned} \tag{12}$$

The index k runs from zero to $(\lambda' + l' + \lambda + l)/2$. The (...) are the $3j$ symbols:

$$\left(\begin{array}{ccc} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{array} \right) = (-1)^{j_3+m_3+2j_1} \frac{1}{\sqrt{2j_3+1}} C^{j_3 m_3}_{j_1 - m_1 j_2 - m_2}.$$

The centrifugal potential is

$$v_{\alpha}^{\lambda l} = \frac{\hbar^2}{m}\Big[\frac{l(l+1)}{x^2} + \frac{\lambda(\lambda+1)}{y^2}\Big], \tag{13}$$

and nucleon-nucleon potentials are $v_{\alpha\alpha'}(x) =< \alpha|v(\mathbf{x})|\alpha' >= \delta_{\lambda\lambda'}\delta_{ss'}\delta_{\sigma\sigma'}\delta_{JJ'}v_{ll'}^{\sigma J}$, where $v_{ll'}^{\sigma J}$ are the potential representatives in the two-body basis $\mathcal{Y}_{l\sigma}^{JJ_z}(\hat{\mathbf{x}})$ (most often abbreviated as ${}^{2\sigma+1}l_J$).

The set of partial differential equation Eqs.(10) must be solved for functions satisfying the regularity conditions

$$\Phi_{\alpha}^{\lambda_0 s_0 M_0}(0,\theta) = \Phi_{\alpha}^{\lambda_0 s_0 M_0}(\rho,0) = \Phi_{\alpha}^{\lambda_0 s_0 M_0}(\rho,\pi/2) = 0. \tag{14}$$

The asymptotic conditions for nd breakup scattering has the following form is given by Eq. (1). To simplify the numerical solution of Eqs.(10), we write down Eqs.(10) in the polar coordinate system ($\rho^2 = x^2 + y^2$ and $\tan\theta = y/x$):

$$\begin{gathered}\Big[E + \frac{\hbar^2}{m}(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{4\rho^2}) - v_{\alpha}^{\lambda l}(\rho,\theta)\Big]U_{\alpha}^{\lambda_0 s_0 M_0}(\rho,\theta) = \\ \sum_{\beta} v_{\alpha\beta}(\rho,\theta)\Big[U_{\beta}^{\lambda_0 s_0 M_0}(\rho,\theta) + \int_{-1}^{1} du \sum_{\gamma} g_{\beta\gamma}(\theta,u,\theta'(\theta,u))U_{\gamma}^{\lambda_0 s_0 M_0}(\rho,\theta')\Big].\end{gathered} \tag{15}$$

Here the first derivative in the radius is eliminated by the substitution $U = \rho^{-1/2}\Phi$. In Eqs. (14),(15) the angular variable θ' is defined by

$$\cos^2\theta'(u,\theta) = \frac{1}{4}\cos^2\theta - \frac{\sqrt{3}}{2}\cos\theta\sin\theta\cdot u + \frac{3}{4}\sin^2\theta. \tag{16}$$

6. Appendix II

To calculate observables for elastic scattering of nucleon from deuteron in the direction $\hat{\mathbf{q}}'$ (initial direction $\hat{\mathbf{q}}$ is along the z-axis), one has to derive the equation for the elastic amplitude as a function of scattering angle. Omitting this derivation, we represent the final expression for this amplitude in MGL basis:

$$\begin{gathered}\hat{a}_{\sigma_z',J_z',\sigma_z,J_z}(\hat{\mathbf{q}}') = \sum_{M}\sum_{\lambda's'}\sum_{\lambda s} \imath^{\lambda-\lambda'}\sqrt{\frac{2\lambda+1}{4\pi}} \\ \times C^{MM_z}_{\lambda' M_z-\sigma_z'-J_z', s'\sigma_z'+J_z'} C^{MM_z}_{\lambda 0, s\sigma_z+J_z} C^{s'\sigma_z'+J_z'}_{1/2\sigma_z',1J_z'} C^{s\sigma_z+J_z}_{1/2\sigma,1J_z} Y_{\lambda' M_z-\sigma_z'-J_z'}(\hat{\mathbf{q}}') a^{M}_{\lambda' s'\lambda s},\end{gathered} \tag{17}$$

with $M_z = \sigma_z + J_z$.

In Eq. (17) $\sigma'\sigma_z'(\sigma,\sigma_z)$ and $J'J_z'(JJ_z)$ are spin and its projection for incoming (scattered) nucleon, and the deuteron in the rest (scattered deuteron), respectively. Thus, the nuclear part of the elastic amplitude is a $(2\times2)\otimes(3\times3)$ matrix in the spin states of nucleon and deuteron, depending on the spherical angles θ and ϕ.

The spin elastic observable formulas can be taken from the review [1]. They are expressed via spin 2×2 matrices σ_i for the nucleon and 3×3 matrices $\mathcal{P}_i$ and $\mathcal{P}_{ik}$ for the deuteron. The latter are related to the deuteron spin matrices S_i:

$$S_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; S_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -\imath & 0 \\ \imath & 0 & -\imath \\ 0 & \imath & 0 \end{pmatrix}, \; S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (18)$$

One has $\mathcal{P}_i = S_i$, $\mathcal{P}_{ik} = 3/2(S_i S_k + S_k S_i)$, $\mathcal{P}_{zz} = 3S_z S_z - 2I$, and $\mathcal{P}_{xx} - \mathcal{P}_{yy} = 3(S_x S_x - S_y S_y)$.

Nucleon analyzing powers A_k are

$$A_k = \frac{\mathrm{Tr}\,(\hat{a}\sigma_k \hat{a}^\dagger)}{\mathrm{Tr}\,(\hat{a}\hat{a}^\dagger)}. \quad (19)$$

If the scattering plane is the xy plane and the y axis points to the direction $\mathbf{q} \times \mathbf{q}'$ then due to parity conservation $A_x = A_z = 0$ and the only non-zero component is A_y. The deuteron vector and tensor analyzing powers are defined as

$$A_k = \frac{\mathrm{Tr}\,(\hat{a}\mathcal{P}_k \hat{a}^\dagger)}{\mathrm{Tr}\,(\hat{a}\hat{a}^\dagger)}, \;\; A_{jk} = \frac{\mathrm{Tr}\,(\hat{a}\mathcal{P}_{jk} \hat{a}^\dagger)}{\mathrm{Tr}\,(\hat{a}\hat{a}^\dagger)}. \quad (20)$$

Parity conservation puts A_x, A_z, A_{xy} and A_{yz} to zero. So the non-vanishing and independent analyzing powers are defined by

$$\imath T_{11} = \frac{\sqrt{3}}{2} A_y, \;\; T_{20} = \frac{1}{\sqrt{2}} A_{zz}, \;\; T_{21} = -\frac{1}{\sqrt{3}} A_{xz}, \;\; T_{22} = \frac{1}{2\sqrt{3}} (A_{xx} - A_{yy}). \quad (21)$$

Also spin transfer coefficients are given in the review [1]. They have the same structure as the quantities above, with slightly different matrices to be inserted between $\hat{a}$ and $\hat{a}^\dagger$.

In the case of nd breakup scattering expression for physical breakup amplitude to calculate breakup observables has much more complex form. Below we present the main details of its derivation. The asymptotic of the wave function with the given incident plane wave is defined as follows

$$\Psi_{q,s_0,s_{0z}}(\mathbf{x},\mathbf{y}) = \frac{e^{\imath KX}}{X^{5/2}} \frac{4\pi}{q} \sum_{\lambda_0,\lambda_{0z},M_0,M_{0z}} c^{\lambda_0,s_0,M_0}_{\lambda_{0z},s_{0z}}(\hat{\mathbf{q}}) \sum_{\alpha} \frac{\mathcal{A}^{\lambda_0 s_0,M_0}_{\alpha}(\theta)}{\sin\theta\cos\theta} Z_\alpha(\hat{\mathbf{y}},\hat{\mathbf{x}}), \quad (22)$$

where the initial plane wave has the form

$$c^{\lambda_0 s_0 M_0}_{\lambda_{0z} s_{0z}}(\hat{\mathbf{q}}) = \imath^{\lambda_0} C^{M_0 M_{0z}}_{\lambda_0 \lambda_{0z} s_0 s_{0z}} [Y_{\lambda_0 \lambda_{0z}}]^*(\hat{\mathbf{q}}), \quad (23)$$

and $Z_\alpha(\hat{\mathbf{x}},\hat{\mathbf{y}})$ are the spin-angular-isospin eigenfunctions.

The exponential factor is multiplied by the function depending only on angles, that are directions of vectors $\mathbf{q}$, $\mathbf{x}$ and $\mathbf{y}$. This gives a probability to find the breakup particles at given directions and so with the amplitude for the breakup with given directions of $\mathbf{p}'$ and $\mathbf{q}'$. So the breakup amplitude is

$$\mathcal{A}_{q,s_0,s_{0z}}(\mathbf{p}',\mathbf{q}') = \frac{4\pi}{q} \sum_{\lambda_0,\lambda_{0z},M_0,M_{0z}} c^{\lambda_0,s_0,M_0}_{\lambda_{0z},s_{0z}}(\hat{\mathbf{q}}) \sum_{\alpha} \frac{\mathcal{A}^{\lambda_0 s_0,M_0}_{\alpha}(\theta)}{\sin\theta\cos\theta} Z_\alpha(\hat{\mathbf{q}}',\hat{\mathbf{p}}'), \tag{24}$$

where now $\theta = \arctan(q'/p')$

The only thing necessary to pass to the formula for the breakup is to project this amplitude onto the state with given projections of spins of the three particles μ_1,μ_2,μ_3. This will obviously be given by introducing into the sum over α in Eq. (24) the projection

$$d^{M_0M_{0z}}_{\alpha,\mu_1\mu_2\mu_3}(\hat{\mathbf{p}}'\hat{,}\mathbf{q}') =< \mu_1\mu_2\mu_3|Z_\alpha(\hat{\mathbf{q}}',\hat{\mathbf{p}}') > . \tag{25}$$

As the result we get the breakup scattering amplitude as a function of final nucleon momenta in the following form

$$\mathcal{A}(\hat{\mathbf{p}}',\hat{\mathbf{q}}',\mu_1\tau_1\mu_2\tau_2\mu_3\tau_3|\mathbf{q},s_0,s_{0z})$$

$$= \frac{4\pi}{q} \sum_{\alpha,\pi,\lambda_0,M_0,\lambda_{0z},M_{0z}} d^{M_0,M_{0z}}_{\mu_1\tau_1\mu_2\tau_2\mu_3\tau_3,\alpha}(\hat{\mathbf{p}}',\hat{\mathbf{q}}') \frac{\mathcal{A}^{M_0}_{\alpha,\lambda_0,s_0}(\theta')}{\sin\theta'\cos\theta'} c^{\lambda_0 s_0 M_0}_{\lambda_{0z},s_{0z}}(\hat{\mathbf{q}}'), \tag{26}$$

where μ_i and τ_i $i = 1,2,3$ are the spin and isospin projections of the three nucleons, $\theta' = \arctan(q'/p')$, and summation goes over $\lambda_{0z} + s_{0z} = M_{0z}$. $\mathcal{A}^{M_0}_{\alpha,\lambda_0,s_0}$ is the spherical inelastic amplitude defined in Eq. (3). The d-coefficients are

$$d^{M_0,M_{0z}}_{\mu_1\tau_1\mu_2\tau_2\mu_3\tau_3,\alpha}(\hat{\mathbf{p}}',\hat{\mathbf{q}}') = (-1)^{\lambda+J+M-1/2}[(2J+1)(2s+1)]^{1/2} C^{\sigma\mu_2+\mu_3}_{\frac12\mu_2,\frac12\mu_3}$$

$$\times C^{t\tau_2+\tau_3}_{\frac12\tau_2,\frac12\tau_3} C^{\frac12-\frac12}_{\frac12-\frac12-\tau_2-\tau_3,t\tau_2+\tau_3} \sum_{LS}[(2L+1)(2S+1)]^{1/2} C^{SS_z}_{\frac12\mu_1,\sigma\mu_2+\mu_3} C^{M_0M_{0z}}_{LM_{0z}-S_z,SS_z}$$

$$\times \left\{ \begin{array}{ccc} l & \sigma & J \\ 1/2 & s & S \end{array} \right\} \left\{ \begin{array}{ccc} \lambda & l & L \\ S & M & s \end{array} \right\} \mathcal{Y}^{L,M_{0z}-S_z}_{\lambda l}(\hat{\mathbf{q}}',\hat{\mathbf{p}}'), \tag{27}$$

where $S_z = \mu_1+\mu_2+\mu_3$ and t=0 or 1 according to antisymmetry condition $l+\sigma+t$ odd.

The breakup differential cross-section is then

$$\frac{d^5\sigma}{d^3p'd^2\hat{\mathbf{q}}'} = \frac{4q'}{3K^3q}|\mathcal{A}(\hat{\mathbf{p}}',\hat{\mathbf{q}}',\mu_1\tau_1\mu_2\tau_2\mu_3\tau_3|\mathbf{q},s_0,s_{0z})|^2. \tag{28}$$

Note that $d^3p'q'd^2\hat{q}' = p'd^2\hat{p}'d^3q'$.

This formula may be transformed to the form which is used by the experimentalists in the lab. system.

$$\frac{d^5\sigma}{dSd^2\hat{\mathbf{k}}_1d^2\hat{\mathbf{k}}_2} = \frac{\sqrt{3}mk_1^2k_2^2}{qK^3\sqrt{D}}|\mathcal{A}(\hat{\mathbf{p}}',\hat{\mathbf{q}}',\mu_1\tau_1\mu_2\tau_2\mu_3\tau_3|\mathbf{q},s_0,s_{0z})|^2, \tag{29}$$

where

$$D = k_1^2\Big(2k_2 - \hat{\mathbf{k}}_2(\mathbf{k}_{lab} - \mathbf{k}_1)\Big)^2 + k_2^2\Big(2k_1 - \hat{\mathbf{k}}_1(\mathbf{k}_{lab} - \mathbf{k}_2)\Big)^2, \tag{30}$$

and S is the arc length along the allowed curve in the $E_1 - E_2$ plane:

$$dS = dE_1\sqrt{1 + \left(\frac{k_2}{k_1}\frac{2k_1 - \hat{\mathbf{k}}_1(\mathbf{k}_{lab} - \mathbf{k}_2)}{2k_2 - \hat{\mathbf{k}}_2(\mathbf{k}_{lab} - \mathbf{k}_1)}\right)^2}. \tag{31}$$

This cross-section refers to the breakup experiment in which all individual spins and isospins of the final nucleons and the spin of the initial neutron-deuteron system are given. The unpolarized cross-section with the isospin projections of the final nucleons given is obtained by summing Eq. (29) over μ_1, μ_2, μ_3 and averaging over s_0 and its projections s_{0z}:

$$\frac{d^5\sigma}{dSd^2\hat{\mathbf{k}}_1 d^2\hat{\mathbf{k}}_2} = \frac{\sqrt{3}mk_1^2k_2^2}{qK^3\sqrt{D}}\frac{1}{6}\sum_{\mu_1,\mu_2,\mu_3,s_0,s_{0z}} |\mathcal{A}(\hat{\mathbf{p}}', \hat{\mathbf{q}}', \mu_1\tau_1\mu_2\tau_2\mu_3\tau_3|\mathbf{q}, s_0, s_{0z})|^2. \tag{32}$$

To write this expression in a more convenient form, one can introduce a matrix between initial and final spins:

$$\hat{\mathcal{A}}(\hat{\mathbf{p}}', \hat{\mathbf{q}}'|\mathbf{q})_{\mu_1,\mu_2,\mu_3|s_0,s_{0z}} \equiv \mathcal{A}(\hat{\mathbf{p}}', \hat{\mathbf{q}}', \mu_1\tau_1\mu_2\tau_2\mu_3\tau_3|\mathbf{q}, s_0, s_{0z}). \tag{33}$$

In terms of this matrix the unpolarised cross-section can be written as

$$\frac{d^5\sigma}{dSd^2\hat{\mathbf{k}}_1 d^2\hat{\mathbf{k}}_2} = \frac{\sqrt{3}mk_1^2k_2^2}{qK^3\sqrt{D}}\frac{1}{6}\mathrm{Tr}\{\hat{\mathcal{A}}^\dagger\hat{\mathcal{A}}\}. \tag{34}$$

Using this matrix one can write all polarization observables in the same form as for the elastic channel. In particular the final proton analyzing power will be given by the formula

$$A_k = \frac{\mathrm{Tr}\{\hat{\mathcal{A}}^\dagger\sigma_k^{(1)}\hat{\mathcal{A}}\}}{\mathrm{Tr}\{\hat{\mathcal{A}}^\dagger\hat{\mathcal{A}}\}}, \tag{35}$$

where $\sigma_{\mu_1'\mu_1}$ is the spin matrix of the 1st nucleon, supposed to be proton ($\tau_1 = +1/2$).

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