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Faddeev calculations for light Ξ-hypernuclei

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Abstract. The hypernuclear systems $NN\Xi$ and $\Xi\Xi N$ are considered as an analogue of nnp (^{3}H) nuclear system (with the notation as AAB system). We use the recently proposed modification for the s-wave Malfliet-Tjon potential. The modification simulates the Extended-Soft-Core model (ESC08c) for baryon-baryon interactions. The ΞN spin/isospin triplet $(S, I) = (1, 1)$ potential generates a bound state with the energy $B_2(AB)$=1.56 MeV. Three-body binding energy B_3 for the states with maximal total isospin is calculated employing the configuration-space Faddeev equations. Comparison with the results obtained within the integral representation for the equations is presented. The different types of the relation between B_2 and $B_3(V_{AA} = 0)$ are discussed.

Keywords: Few-body systems, Hypernuclei, Nuclear forces, Faddeev equations

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1. Introduction

The first Ξ-hypernuclear bound state has been reported in Ref. [1]. The lifetime of a Ξ-hypernucleus is long enough to enable the hypernuclear state to be well defined. According the current experimental data, the Ξ-nucleus interactions are attractive [2]. In particular, the hyprnucleus $^{12}_{\Xi}$Be can be interpreted by assuming a nucleus Wood-Saxon potential with a strength parameter of about -14 MeV [3]. Another hypernucleus $^{15}_{\Xi}$C is considered to be the cluster system ^{14}N(ground state) + Ξ, where Ξ can be in s or p-wave state [4].

The stable states in the systems ΞNN and $\Xi\Xi N$ were recently predicted in Refs. [5, 6, 7] based on the recent update of the Extended-Soft-Core (ESC08c) model [8, 9, 10] for baryon-baryon interactions. This model has predicted the ΞN bound spin/isospin triplet $(S, I) = (1, 1)$ state with three-body energy B_3 to be equal to 1.56 MeV. This bound state of proton and Ξ^0 or neutron and Ξ^- has maximal isospin of the ΞN pair. For the three-body systems when all pairs NN, ΞN, and $\Xi\Xi$ are in triplet isospin states, the strong decay $N\Xi \to \Lambda\Lambda$ is forbidden. Such three-body systems can be stable under the strong interaction. The first calculations [6, 7] based on the assumption yield the existence of bound states for the ΞNN and $\Xi\Xi N$ systems.

In the presented work, we use the differential Faddeev equations to mathematically formulate the bound state problems for the ΞNN and $\Xi\Xi N$ systems. The alternative treatment is presented in Refs. [6, 7] where the integral Faddeev equations were applied. Our calculations for the systems are generally in the agreement with the results [6, 7]. However, we found that a small correction for the results is needed. We present our results along with the correction [11] of the results published in Refs. [6, 7]. Additionally, the binding energy for the spin, isospin (0, 1) bound state for the $\Xi\Xi\alpha$ system is calculated. This state was not considered in Refs. [6, 7].

The models for ΞNN and $\Xi\Xi N$ ($\Xi\Xi\alpha$) are restricted by the s-wave approach. The coupling to higher-mass channels, ΣΛ and ΣΣ, does not taken into account assuming that their contributions have the second order of smallness to the binding energy of three-body system. The calculations do not also take into account the Coulomb force.

2. Formalism

2.1 Faddeev equations for AAB system

The differential Faddeev equations [12] can be reduced to a simpler form for the case of two identical particles (like an AAB system). In this case the total wave function of the system is decomposed into the sum of the Faddeev components U and W corresponding to the $(AA)B$ and $(AB)B$ types of rearrangements: $\Psi = U + W \pm PW$, where P is the permutation operator for two identical particles. In the latter expression the sign "+" corresponds to two identical bosons, while the sign "−" corresponds to two identical fermions, respectively. The set of the Faddeev equations is written as following:

$$\begin{aligned}(H_0 + V_{AA} - E)U &= -V_{AA}(W \pm PW),\\ (H_0 + V_{AB} - E)W &= -V_{AB}(U \pm PW).\end{aligned} \tag{1}$$

Here, H_0 is the operator of kinetic energy of the Hamiltonian taken for corresponding Jacobi coordinates. The functions V_{AA} and V_{AB} describe the pair interactions between the particles. The model space is restricted to the states with the total angular momentum $L = 0$, the momentum of pair $l = 0$, and momentum $\lambda = 0$ of the third particle respectively to the center of mass of the pair.

2.2 s-wave approach

The description of the above mentioned AAB systems is distinguished by the masses of particles and the type of AA and AB interactions. We use s-wave V_{AA} and V_{AB} potentials, which are spin-isospin dependent. This requires to write Eq. (1) with the corresponding spin-isospin configurations.

The separation of spin-isospin variables leads to the Faddeev equations for the considering systems in the following form:

$$\begin{aligned}(H_0 + V_{AA} - E)\mathcal{U} &= -V_{AA}D(1+p)\mathcal{W} \ ,\\ (H_0 + V_{AB} - E)\mathcal{W} &= -V_{AB}(D^T\mathcal{U} + Gp\mathcal{W}) \ ,\end{aligned} \tag{2}$$

where matrices D and G are defined by the nuclear system under consideration, $\mathcal{W}$ is a column matrix with the singlet and triplet parts of the W component of the wave function of a nuclear system, and the exchange operator p acts on the coordinates of identical particles.

For the ^{3}H nucleus, considered as pnn system in the state (S, I)=$(1/2, -)$, we applied the isospin-less approach proposed in Ref. [13]. The inputs into Eq. (2) are the following: the spin singlet nn potential $V_{AA} = v^s_{nn}$ and $V_{AB} = diag\{v^s_{np}, v^t_{np}\}$ that is a diagonal 2×2 matrix with the spin singlet v^s_{np} and spin triplet v^t_{np} np potentials, respectively, and

$$D = (-\frac{1}{2}, \frac{\sqrt{3}}{2}), \quad G = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}, \quad \mathcal{W} = \begin{pmatrix} \mathcal{W}^s \\ \mathcal{W}^t \end{pmatrix}, \quad \mathcal{U} = \mathcal{U}^s, \tag{3}$$

where $\mathcal{W}^s$ and $\mathcal{W}^t$ are the spin singlet and spin triplet parts of the $\mathcal{W}$ component. Within the isospin formalism when the protons and neutron are identical particles, instead of Eq. (2), which is a set of three equations, one has the set of two equations for the state (S, I)=$(1/2, 1/2)$ of the three nucleon system NNN:

$$(H_0 + V_{NN} - E)\Phi = -V_{NN}B(p^+ + p^-)\Phi \ , \tag{4}$$

where

$$V_{NN} = diag\{v^s_{NN}, v^t_{NN}\}, \qquad B = \begin{pmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \Phi^s \\ \Phi^t \end{pmatrix}$$

and $p^{\pm}$ are the operators of cyclical permutations for coordinates of the particles.

2.3 Spin-isospin configurations

In Eq. (1), the Faddeev component U (and W) of the total wave-function is expressed in terms of spin and isospin:

$$U = \mathcal{U}\chi_{spin}\eta_{isospin}.$$

The graphical representation of the spin-isospin configurations in the $\Xi\Xi N$ and $NN\Xi$ systems is given in Fig. 1. Here, we have taken into account that the spin

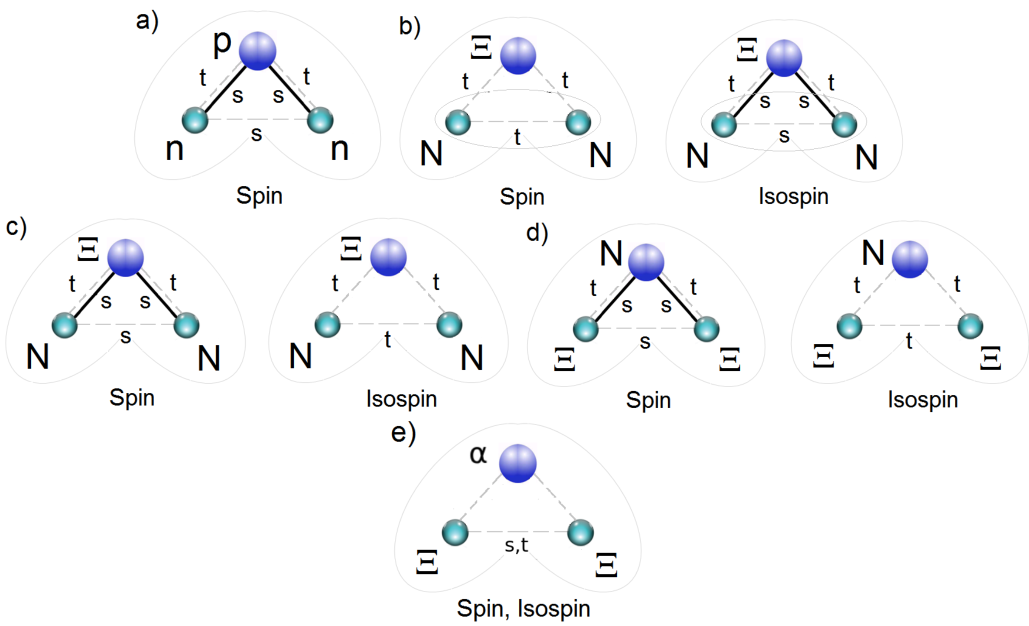

Figure 1: The spin-isospin configurations in AAB systems: a) nnp, (S,I)= $(1/2,-)$, b) $NN\Xi$, (S,I)=$(3/2,3/2)$, c) $NN\Xi$, (S,I)=$(1/2,3/2)$, d) $\Xi\Xi N$, (S,I)=$(1/2,3/2)$, e) $\Xi\Xi\alpha$, (S,I)=$(0,1)$. The pair potentials have spin or isospin singlet and triplet components (noted as s and t). The two-body bound states are noted by ovals.

(isospin) basis of the spin (isospin) 3/2 state for three-body AAB system is formed by a single basis element. Thus, the Faddeev equations for each system considered have the form (2)-(3). The equation for the state $\Xi\Xi\alpha$ (S,I)=$(0,1)$ has a "scalar form" instead the form (2)-(3):

$$\begin{aligned}(H_0 + V_{AA} - E)\mathcal{U} &= -V_{AA}(1+p)\mathcal{W},\\ (H_0 + V_{AB} - E)\mathcal{W} &= -V_{AB}(\mathcal{U} + p\mathcal{W}),\end{aligned} \quad (5)$$

where the V_{AB} and $\mathcal{W}$ are scalars: $V_{AB} = v_{AB}$. Here, we used what the spin-isospin part of the wave function of the fermion pair $\Xi\Xi$ is antisymmetric relatively to the permutation P in Eq. (1).

Let us assume that $V_{AA} = 0$, then Eq. (5) is reduced to a single equation:

$$(H_0 + V_{AB} - E)\mathcal{W} = -V_{AB}p\mathcal{W}. \quad (6)$$

This equation is similar to one obtained in Ref. [6] for the $NN\Xi$, (S,I)=$(3/2,3/2)$ state within the integral Faddeev equations. However, the right hand side of Eq. (6) has opposite sign. The restriction $V_{AA}=0$ corresponds to the situation when NN potential can be neglected for the spin/isospin triplet $(S,I)=(1,1)$ state. The differential Eq. (6) shows that the right hand side term is attractive (including attractive $\Xi\alpha$ potential) and can give additional contribution into the binding energy coming from the left hand side term. The corresponding term for the $NN\Xi$, (S,I)=$(3/2,3/2)$ state is repulsive due to symmetry of 3/2 spin/isospin basis functions relatively permutation of two identical particles that holds the sign "minus" before the operator P in Eq. (1). The state $NN\Xi$, (S,I)=$(3/2,3/2)$ is unbound [6].

2.4 Interactions

In this section we consider the two-body interactions, which are the inputs to our present study. To describe a nucleon-nucleon, we use the semi-realistic Malfliet and Tjon MT I-III [14] potential with the modification from Ref. [15]. The MT I-III model has the Yukawa-type form:

$S=0$, $I=1$:

$$V_{NN}(r)=(-513.968exp(-1.55r)+1438.72exp(-3.11r))/r,$$

$S=1$, $I=0$:

$$V_{NN}(r)=(-626.885exp(-1.55r)+1438.72exp(-3.11r))/r,$$

where the strength parameters are given in MeV and range parameters are given in fm^{-1}. The parameters were chosen in Ref. [14] to reproduce the experimental data for np-scattering. It has to be noted that we do not use isospin formalism for the nnp system. Thus, the protons and neutrons are not identical. The details of such treatment are presented in Ref. [13]. To take into account that the nn interaction is not equivalent to np interaction (that is known as the charge dependence of NN interaction), we have made modification of the spin singlet $(S,I)=(0,1)$ component of the MT I-III potential according Ref. [13] and have defined spin singlet nn potential. The modification was performed by scaling strength parameter. The scaling parameter γ is fixed as γ=0.975 to reproduce experimental nn scattering length for which we used the value of -18.8 fm [16, 17]. By this way, we have obtained three potentials v^s_{nn}, v^s_{np} and v^t_{np} needed for Eq. (2). Note that the MT I-III potential is not defined for the spin/isospin triplet $(S,I)=(1,1)$ and singlet $(S,I)=(0,0)$ states. The corresponding potentials are taken to be equal zero.

The ΞN and $\Xi\Xi$ potentials simulating the ESC08c Nijmegen model are written in the form [7]:

$S=0$, $I=1$:

$$V_{\Xi N}(r)=(-290.0exp(-3.05r)+155.0exp(-1.6r))/r,$$

$S = 1$, $I = 0$:

$$V_{\Xi N}(r) = (-568.0exp(-4.56r) + 425.0exp(-6.73r))/r,$$

$S = 0$, $I = 1$:

$$V_{\Xi\Xi}(r) = (-155.0exp(-1.75r) + 490.0exp(-5.6r))/r.$$

The parameters of the potentials were fixed to reproduce the scattering lengths and effective radii given by the ESC08c Nijmegen model for the baryon-baryon interaction [8, 9, 10].

For the $\Xi\alpha$ interaction we use the Isle-type potential [18] which has the Gaussian form:

$$V_{\Xi\alpha}(r) = 450.4exp(-(r/1.269)^2) - 404.9exp(-(r/1.41)^2).$$

with parameters from Ref. [19].

3. Numerical results

The ground state binding energies B_3 of the NNN, nnp, $NN\Xi$, $\Xi\Xi N$, $\Xi\Xi\alpha$ systems were calculated using the models suggested above. The numerical results are presented in Table 1. For each system, we show the spin-isospin state (S, I) and two-body energies $B_2(AA)$ and $B_2(AB)$ for AA and AB pairs. Additionally, we present the three-body binding energy calculated under the condition $V_{AA} = 0$. Our results are compared with ones obtained within the integral representation of Refs. [7, 11]. One can see that results of both approaches are in the agreement with high accuracy.

Table 1: Binding energy B_3 (in MeV) calculated for various systems AAB within the differential (DFE) and integral Faddeev (IFE) equations. The $B_3(V_{AA} = 0)$ is shown in brackets. Binding energy B_2 (in MeV) for AA and AB pairs are also presented. Here m_N=938.91 MeV, m_Ξ=1318.07 MeV, m_α=3727.38 MeV.

System	(Spin, Isospin)	$B_2(AA)$	$B_2(AB)$	B_3, DFE	B_3, IFE[7, 11]
NNN	(1/2, 1/2)	2.23	–	8.58 [13]	–
nnp	(1/2, –)	–	2.23	8.38[13] (3.40)	–
$NN\Xi$	(3/2, 1/2)	2.23	1.67	17.205 (2.213)	17.203
$NN\Xi$	(1/2, 3/2)	–	1.67	2.886 (1.785)	2.8855
$\Xi\Xi N$	(1/2, 3/2)	–	1.67	4.512 (3.408)	4.5119
$\Xi\Xi\alpha$	(0, 1)	–	2.09	7.635 (4.335)	–

The "spin/isospin complication" [20] of the Faddeev equations for the considered systems is appeared by the matrix form of Eq. (3) and leads to the following evaluation for the three-body binding energy of the $NN\Xi$ system in the spin-isospin states (S,I)=$(3/2,1/2)$, $(1/2,3/2)$:

$$B_2(AB) < B_3(V_{AA}=0) < 2B_2(AB). \tag{7}$$

The value of $B_3(V_{AA}=0)$ is restricted by 3.34 MeV. The similar result we have for the nnp system. In this case, $B_3(V_{AA}=0)$ is restricted by 4.46 MeV. In contrast, the scalar form (5) of Eq. (2) for the case $\Xi\Xi\alpha$ (S,I)=$(0,1)$ leads to the relation:

$$B_3(V_{AA}=0) > 2B_2(AB). \tag{8}$$

This relation is known as the mass polarization effect which takes place when $m_B/m_A > 1$ [21, 22]. For the spin-isospin state (S,I)=$(0,1)$ of $\Xi\Xi\alpha$ system, the mass polarization energy can be evaluated [20]. The contribution of this energy $(B_3(V_{AA}=0)-2B_2(AB))/B_3(V_{AA}=0)$ in the three-body bound energy is equal 3.6% that is compatible with the values of 2%-4% [22, 20] for the similar nuclear system $\Lambda\Lambda\alpha$. The similarity takes place due to approximate equality of the masses of non-identical particles: $m_B/m_A\sim 3$ for the $\Xi\Xi\alpha$ and $m_B/m_A\sim 3$ for $\Lambda\Lambda\alpha$. In the limit $m_B/m_A >> 1$ the mass polarization effect can be neglected and $B_3(V_{AA}=0)=2B_2(AB)$. The case $m_B/m_A<1$ is realized for the system $\Xi\Xi N$ $(1/2,3/2)$. There is no a simple relation between $B_3(V_{AA}=0)$ and $2B_2(AB)$ for the case. One can define the incremental binding energy $\Delta B_{\Xi\Xi}$ for the system $\Xi\Xi N$ $(1/2,3/2)$ as $\Delta B_{\Xi\Xi}=B_3-2B_2(\Xi N)$ according the analogue with the ${}^{6}_{\Lambda\Lambda}$He hypernucleus. For ${}^{6}_{\Lambda\Lambda}$He, the incremental binding energy is defined as $\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}\left({}^{A}_{\Lambda\Lambda}Z\right)-2B_{\Lambda}\left({}^{(A-1)}_{\Lambda}Z\right)$ [21, 22]. Calculating the incremental energy, one can evaluate the strength of the $\Lambda\Lambda$ interaction. For the system $\Xi\Xi N$ $(1/2,3/2)$, the energy includes significant contribution of the mass polarization energy because $m_N/m_\Xi\sim 1$. Regardless that the relation (7) is not satisfied, the more appropriate value for an evaluation of the strength of the $\Xi\Xi$ interaction in $\Xi\Xi N$ $(1/2,3/2)$ is the value of $B_3-B_3(V_{\Xi\Xi}=0)$. The corresponding evaluation can be obtained from Table 1. The $\Xi\Xi$ interaction is attractive in $\Xi\Xi N$ $(1/2,3/2)$. Analogically, the spin singlet NN interaction is also attractive in the $NN\Xi$ $(1/2,3/2)$ system. These attractive forces add about 1 Mev to the binding energies of the mirror systems. Thus, the matrix elements $<\Psi|V_{AA}|\Psi>$ have the close values for the systems. It is possible, because the $\Xi\Xi N$ system is more compact (larger B_3 value) and the $\Xi\Xi$ potential has a minimum closer to the origin as is shown in Fig. 2. The Faddeev components U, $\mathcal{W}$ for the $NN\Xi$ $(1/2,3/2)$ and $\Xi\Xi N$ $(1/2,3/2)$ systems are presented in Fig. 3. From the figure, one can see that the system $\Xi\Xi N$ $(1/2,3/2)$ is more compact than the $NN\Xi$ $(1/2,3/2)$ system. For both systems, the rearrangement channel $A+(AB)$ dominates due to existence of the isospin singlet ΞN bound state.

The mirror $NN\Xi$ $(1/2,3/2)$ and $\Xi\Xi N$ $(1/2,3/2)$ systems under the condition $V_{AA}=0$ can be transformed "one into another" by changing the particle masses. The parameter $\xi\geq 0$ sets this transformation by the formula: $m_A^\xi=(1+\xi)m_A$, $m_B^\xi=(1-\xi m_A/m_B)m_B$. The results of calculations for $2E_2$ and $E_3(V_{AA}=0)$

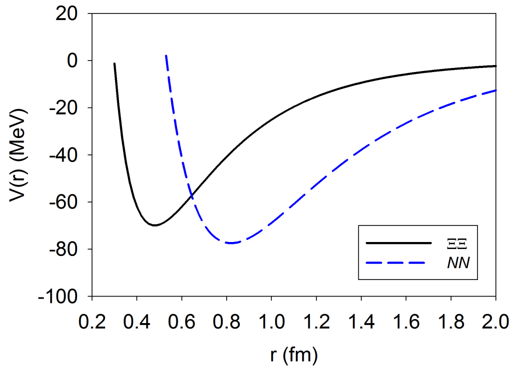

Figure 2: The NN (S,I)=(0,1) and $\Xi\Xi$ (S,I)=(0,1) potentials.

as a function of ξ are shown in Fig. 4a). The transformation $NN\Xi$ $(1/2,3/2)$ to $\Xi\Xi N$ $(1/2,3/2)$ replaces the ratio $m_B/m_A > 1$ to the ratio $m_B/m_A < 1$. One can see that the relation (7) is well satisfied up to ξ=0.2 when $m^{\xi}_B/m^{\xi}_A \geq 1$. We conclude that the relation (7) is not guaranteed when $m^{\xi}_B/m^{\xi}_A \ll 1$. The affect of the AB potential on the relation (7) is obvious. To show this we have repeated the calculations for more deep spin triplet $N\Xi$ potential. The potential has been scaled by the factor of 1.05. The result is shown in Fig. 4b). The relation (7) is satisfied for all possible values ξ for this case.

It has to be noted that, as follows from Table 1 for the $NN\Xi$ $(3/2,1/2)$ state, the three-body system having two bound subsystems has a deep bound state. The value of this $NN\Xi$ $(3/2,1/2)$ binding energy is related with two-body energies as $B_3 >> 2B_2(AB) + B_2(AA)$. Obviously, the V_{AA} potential plays a key role for formation of the bound state. We assume that it is a general property of such three-body systems.

4. Conclusions

We studied the hypernuclear system $NN\Xi$ (and $\Xi\Xi N$) based on the configuration-space Faddeev equations. The baryon-baryon potential of ESC08c model, which generates the ΞN $(S,I) = (1,1)$ s-wave bound state, results in the stable states for these three-body systems. The stability relatively $N\Xi \to \Lambda\Lambda$ conversion is provided by fixing the states with maximal isospin. Our results and ones obtained within the integral Faddeev equation formalism [7, 11] are in agreement with high accuracy. Additionally, we have calculated the binding energy of the $\Xi\Xi\alpha$ $(S,I) = (0,1)$ state. The relations between B_2 and $B_3(V_{AA} = 0)$ were proposed for the "spin/isospin complicated" and "scalar" states. The corresponding relations are significantly different.

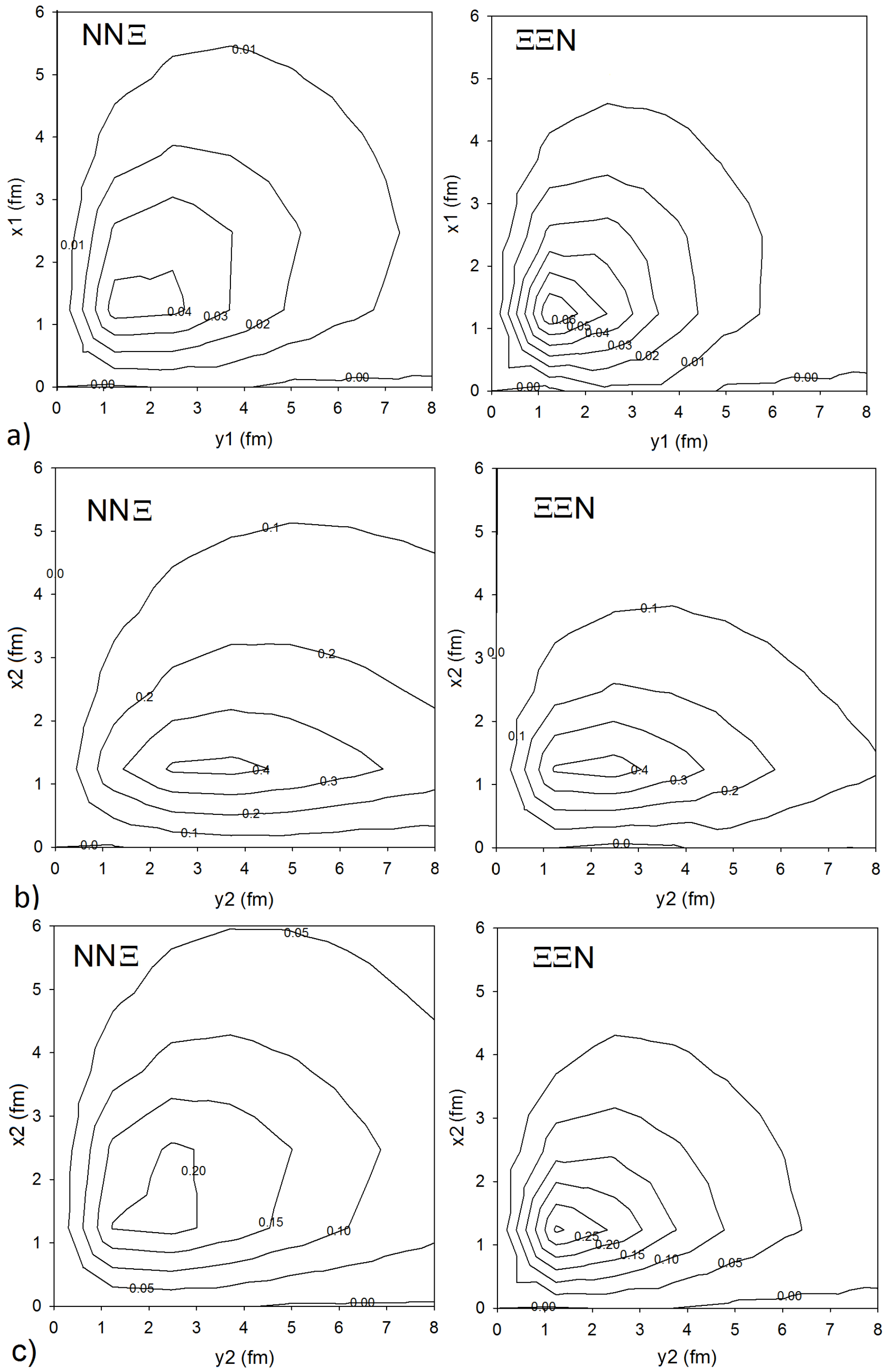

Figure 3: The contour plots of the Faddeev components $\mathcal{U}$ a) and $\mathcal{W}$ b),c) for the $NN\Xi$ $(1/2, 3/2)$ (Left) and $\Xi\Xi N$ $(1/2, 3/2)$ (Right) bound states. The Jacobi coordinates corresponding to the components $\mathcal{U}$ and $\mathcal{W}$ are presented as $x1, y1$ and $x2, y2$.

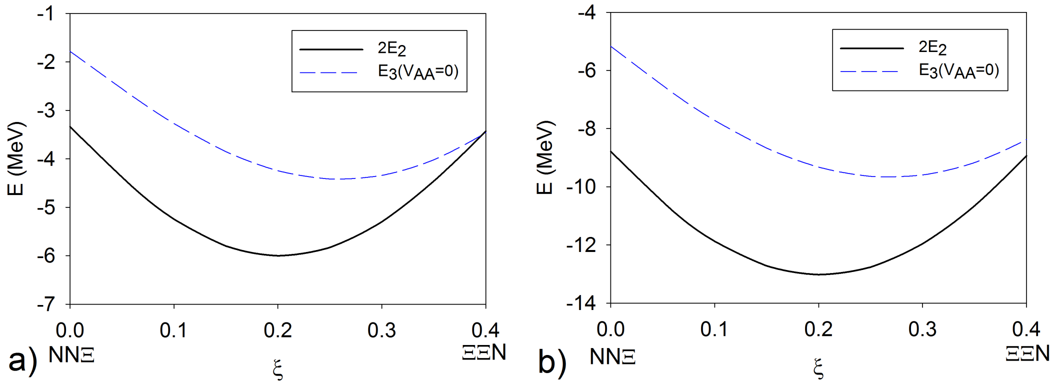

Figure 4: The transformation $NN\Xi$ $(1/2, 3/2)$ to $\Xi\Xi N$ $(1/2, 3/2)$ when $V_{AA} = 0$. The $2E_2$ (solid line) and $E_3(V_{AA} = 0)$ (dashed line) as a function of ξ are shown. The parameter ξ is related to the $NN\Xi$ $(1/2, 3/2)$ system, when ξ=0, and - to the $\Xi\Xi N$ $(1/2, 3/2)$ system, when ξ=0.4. a) The original spin triplet $N\Xi$ potential is used. b) The spin triplet $N\Xi$ potential is scaled by 1.05.

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