



Guided Modes of a Planar Gradient Waveguide

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Abstract. The mathematical model of light propagation in a planar gradient optical waveguide consists of the Maxwell's equations supplemented by the matter equations and boundary conditions. In the coordinates adapted to the waveguide geometry, the Maxwell's equations are separated into two independent sets for the TE and TM polarizations. Each of the systems can be transformed to a second order ordinary differential equation. The boundary conditions for Maxwell's equations are reduced to two pairs of boundary conditions for the obtained equations. Thus, the problem of describing a complete set of modes in a regular planar waveguide is formulated in terms of an eigenvalue problem. For each polarization there are three types of waveguide modes: guided modes, substrate radiation modes, and cover radiation modes. In this work we implement the numerical-analytical calculation of all types of waveguide modes.

Keywords: waveguide propagation of electromagnetic radiation, equations of waveguide modes of regular waveguide, complete set of modes of a planar gradient waveguide

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