



## How the Oblate Spheroidal Coordinates Facilitate Modeling of Quantum Rings

V. A. Roudnev<sup>1,a</sup>, A. M. Puchkov<sup>1</sup> and A. V. Kozhedub<sup>1</sup>

<sup>1</sup> Physics Department, St. Petersburg State University, Universitetskaya nab. 7-9, St. Petersburg, 199034 Russia

**e-mail:** <sup>a</sup> v.rudnev@spbu.ru

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**Abstract.** We propose a new approach for modeling three-dimensional single particle quantum rings based on separation of variables in oblate spheroidal coordinates. This approach has an advantage in the variety of the ring cross sections that can be modeled with high computational efficiency. We illustrate this by studying the shape dependence of the energy spectrum for a single particle confined to the ring of triangular cross section. The spectrum can demonstrate parabolic or non-parabolic behavior as a function of the magnetic quantum number depending on the ring profile.

**Keywords:** quantum rings, spectrum, shape

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