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## **A fractional-order tuberculosis model with fast and slow progression**

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**Abstract.** In this paper, we introduce a fractional-order tuberculosis model with fast and slow progression. We show the existence of non-negative solutions of the model, and also give a detailed stability analysis of the disease-free and endemic equilibria. Numerical simulations are presented to illustrate the results.

**Keywords:** fractional order, tuberculosis model, stability, predictor-corrector method

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