The regression analysis as the tool to determine the mathematical model of dynamic stress of the conveyor belt

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Abstract. The belt conveyor system belongs to the continuous transport systems used in various industries. In terms of the belt conveyor operation, a crucial part of a belt conveyor is a conveyor belt. During the operation, the conveyor belt is exposed to the impact of several stresses of various types that result in the belt wearing and aging process. One of the main reasons why a conveyor belt is damaged, and, as frequently is a case, its service life thus terminates, is the dynamic stress. The aim of this article is to determine the relationship between the impact force or the tension force on one side and the weight of material falling onto the rubber-fabric conveyor belt and the impact height on the other side, based on the data obtained from the experimental research.

Keywords: conveyor belt, dynamic stress, impact force, tension force, regression model
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1. Introduction

In the last years, the belt conveyors belong to the most frequently used means of transport in various industries. Due to their advantages, they are used in various operating conditions, particularly in the continuous transportation of bulk materials. The key structural component of a belt conveyor, from the operation point of view, is the conveyor belt. Belt conveyor is a limited range, continuously moving transport facility that carries material on the belt surface, between two belts or inside a belt [1], [2].

The basic structural components of a conveyor belt are the carcass and the cover layers (Figure 1). The carcass provides sufficient strength and a conveyor belt resistance to impact. The cover layers (upper and lower) serve as the protection against mechanical damage, against humidity, and against chemical and thermal effects to which a conveyor belt is exposed. Depending on the carcass type, we distinguish between rubber-fabric and steel cord conveyor belts.

![Figure 1: The structure of the rubber-textile conveyor belt.](image)

On one hand, the conveyor belt is one of the crucial elements of the belt conveyor system; on the other hand, it is also one of the weakest and the most expensive elements [3]. The damage thereto often results in unplanned downtime in the operation. The factors causing damage to a conveyor belt include, for example, conveyor belt parameters and properties, type of transported material, and overall transportation conditions. The damage is also manifested, for example, on the upper cover layer, in form of transversal and longitudinal scratches and punctures. They may even result in the damage to the upper or lower cover layers, or to the carcass of the conveyor belt.

The practical experience indicates that the most critical site, where a high percentage of all conveyor belt damage types occur, is the so-called chutes [2], [4]. Due to the material falling onto the conveyor belt, the point impact load is formed at the chutes, representing one of the main reasons of the conveyor belt damage. The objective of the ongoing research is the monitoring and the determination of the effects of the dynamic load induced by the material falling onto the conveyor belt.

2. The testing equipment

The testing equipment for the puncture strength testing of conveyor belts was designed and structured at the Institute of Industrial Logistics and Transport of the
Technical University in Kosice (Figure 2). The equipment comprises a hydraulic system for fastening the belt specimen and an additional hydraulic system for fastening the specimen during the test. The structure of the testing equipment is based on the current requirements resulting from the previous research, as well as on the requirements of conveyor belt manufacturer [5].

Figure 2: The testing equipment.

By changing the weight of the drop hammer, it is possible to simulate various specific weights of the falling material. Changing a drop hammer’s head simulates various types of falling material. By changing the impact height of the drop hammer, various operational impact heights of the material falling onto the conveyor belt are simulated. The choice of a conveyor belt type is based on the operating conditions and requirements.

3. Regression and correlation analysis

The relationship between variable and several independent variables \( x_j \), where \( j = 1, 2, \ldots, k \) can be expressed by the multiple linear regression model in the form

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon,
\]

where \( \beta_0 \) and \( \beta_j \) for \( j = 1, 2, \ldots, k \) are parameters of regression model and \( \epsilon \) is a random error. The point estimate of the multiple linear regression model is the regression function

\[
Y = b_0 + b_1 x_1 + b_2 x_2 + \cdots + b_k x_k,
\]

where \( Y \) is theoretical (estimated) value of the dependent variable, \( b_0 \) and \( b_k \) are the estimated parameters \( \beta_0 \) and \( \beta_j \) for \( j = 1, 2, \ldots, k \). The parameters of the regression model are estimated using the method of least squares [6]. To verify the statistical significance of the regression model we use the F-test the statistical
significance of the model. The null and alternative hypotheses are $H_0$: regression model is not statistically significant against $H_1$: regression model is statistically significant. The test statistics is computed using the form

$$F = \frac{(n - k - 1) \sum_{i=1}^{n} (Y_i - \bar{y})^2}{k \sum_{i=1}^{n} (y_i - Y_i)^2},$$

where $y_i$ is the $i$-th observed value of dependent variable, $Y_i$ is the $i$-th estimated value of dependent variable using the regression model, $\bar{y}$ mean of the dependent variable, $k$ number of independent variables and $n$ is the number of observations.

If the null hypothesis is true, the test statistics $F$ is an observed value of an $F$ distributed random variable with $k$ and $(n - k - 1)$ degrees of freedom. The null hypothesis is rejected at the level of significance $\alpha$, if $F > F_{1-\alpha}(k, n - k - 1)$. In case of rejection of the null hypothesis at least one explanatory variable has a statistically significant effect on the test explained variable.

The statistical significance of the regression model parameters will be verified by testing the statistical significance of regression parameters $\beta_j$. The null and alternative hypotheses are: Parameter of regression model is not statistically significant against the alternative hypothesis: Parameter of regression model is statistically significant. The test statistics computed using the form

$$t = \frac{b_j}{s_{b_j}},$$

where $b_j$ is the point estimate of the parameter $\beta_j$ and $s_{b_j}$ is the estimated standard error. We reject the null hypothesis at the level of significance $\alpha$, if the inequality $|t| > t_{1-\alpha}(n - k - 1)$ holds.

In hypothesis testing acceptance or rejection of the null hypothesis can be carried also by decision rule for a $p$-value. If the $p$-value is less than the level of significance $\alpha$ the null hypothesis is rejected. If the $p$-value is greater than the level of significance $\alpha$, the null hypothesis is accepted.

4. Realization of the experiment

Experimental research is carried out in order to identify real values of the impact force and the tension force within the operating conditions simulation and the comparison thereof with the results obtained using regression models. The experiment was carried out with a rubber-fabric conveyor belt P 1600 with the strength of 1600 N·mm$^{-1}$.

Test specimen preparation method is described in [7]. A test object was fixed at both ends in hydraulically operated clamps. Another hydraulic device was used to stretch it, applying the force equal to 1/10 of the belt strength specified by the
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Figure 3: The drop-hammer-sphere impactor.

Figure 4: Damage of the conveyor belt-puncture.

Figure 5: Impact and tension force \((m = 60 \text{ kg})\).
manufacturer. In the experiment we used the drop-hammer with sphere impactor (Figure 3).

The drop-hammer of the relevant weight (from 50 kg to 90 kg with 10 kg difference) is lifted by a pulley block to the required height (from 0.4 m to 2.6 m, 0.2 m difference), from which it is released by free fall onto the conveyor belt. During which values of the tension and impact force [kN] units are recorded during the whole measurement by two tens-meter sensors. Evaluation of tests in case of puncture detection consists in a visual inspection of conveyor belt (Figure 4).

The plot in Figure 5 shows the development of the impact force (resp. tension force) and the height of the drop hammer in time for measurements on the textile-rubber conveyor belt with a 60 kg drop hammer dropped.

5. Results

5.1 Regression model

The aim of the experiment is to determine the dependence of impact force $F_I$, respectively tension force $F_T$ from independent variables Weight of the ram ($m$) and The amount of impact ram ($h$). We appealed to the basic model by [7]

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \epsilon,$$

where

$$x_1 = m, x_2 = h, x_3 = m^2, x_4 = mh, x_5 = h^2$$

and $F_I$, resp. $F_T$. The point estimate model has the form

$$y = b_0 + b_1 m + b_2 h + b_3 m^2 + b_4 mh + b_5 h^2.$$ 

Impact force

Point estimate regression model that captures the dependence of impact force from selected independent variables has the form

$$F_I = 2.364 - 0.0533m + 1.9834h + 0.0008m^2 + 0.1053mh - 0.6823h^2.$$ 

To check the statistical significance of the model, we used the $F$-test of the statistical significance of the model ($\alpha = 0.05$). Because $p$-value $= 4 \cdot 10^{-85} \ll \alpha$, we assume that the proposed regression model is statistically significant. Statistical significance of each parameter regression model has been verified through a test of statistical significance of the regression parameter. The results show that all of the parameters appear to be statistically significant (Table 1).

Multiple index value determination is $I^2 = 0.999$, which means that 99.9% of the variability of the variable $F_I$ can be explained by the influence of the variables Weight of the ram, The amount of impact ram and their interaction.
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Parameter | The point estimate | The lower limit of 95% of the estimate | The upper limit of 95% of the estimate | \( p \)-value | Statistical significance of the parameter
---|---|---|---|---|---
\( b_0 \) | 2.3641 | 0.8470 | 3.8812 | 0.0028 < \( \alpha \) | significant
\( b_1 (m) \) | -0.0533 | -0.0962 | -0.0962 | 0.0158 < \( \alpha \) | significant
\( b_2 (h) \) | 1.9834 | 1.5304 | 1.5304 | 3.7E-12 < \( \alpha \) | significant
\( b_3 (m^2) \) | 0.0008 | 0.0005 | 0.0005 | 8.9E-7 < \( \alpha \) | significant
\( b_4 (mh) \) | 0.1053 | 0.1002 | 0.1002 | 1.6E-44 < \( \alpha \) | significant
\( b_5 (h^2) \) | -0.6823 | -0.7890 | -0.7890 | 2.1E-18 < \( \alpha \) | significant

Table 1: Estimates of the parameters of a regression model – impact force.

**Tension force**

Point estimate regression model that captures the dependence of tension force from selected independent variables has the form

\[
F_T = 21.0435 + 0.3026m + 9.6174h - 0.0011m^2 + 0.1404mh - 2.0345h^2.
\]

To check the statistical significance of the model, we used the \( F \)-test of the statistical significance of the model (\( \alpha = 0.05 \)). Because \( p \)-value = 4 \times 10^{-71} \ll \alpha \), we assume that the proposed regression model is statistically significant. The results show that all of the parameters appear to be statistically significant (Table 2).

Parameter | The point estimate | The lower limit of 95% of the estimate | The upper limit of 95% of the estimate | \( p \)-value | Statistical significance of the parameter
---|---|---|---|---|---
\( b_0 \) | 21.0435 | 16.3096 | 25.7775 | 2.2E-12 < \( \alpha \) | significant
\( b_1 (h^2) \) | 0.3026 | 0.1689 | 0.4362 | 3.0E-5 < \( \alpha \) | significant
\( b_2 (m) \) | 9.6174 | 8.1852 | 11.0196 | 2.5E-19 < \( \alpha \) | significant
\( b_3 (h) \) | -0.0011 | -0.0021 | -0.0002 | 0.0191 < \( \alpha \) | significant
\( b_3 (m^2) \) | 0.1404 | 0.1253 | 0.1555 | 6.6E-26 < \( \alpha \) | significant
\( b_4 (mh) \) | -2.0347 | -2.3618 | -1.7075 | 6.7E-18 < \( \alpha \) | significant

Table 2: Estimates of the parameters of a regression model – tension force.

The index of the determination is \( I^2 = 0.997 \), which means that 99.7% of the variability of the variable \( F_T \) can be explained by the influence of the variables *Weight of the ram*, *The amount of impact ram* and their interaction.

Graphical representation of the experimental (measured) values and theoretical (calculated) values of impact force and tension force to the belt P 1600 is on Figure 6 and Figure 7.
Figure 6: Experimental and theoretical values of impact force.

Figure 7: Experimental and theoretical values of tension force.
6. Conclusions

Within the modelling and experimental research, a single type of a rubber-fabric conveyor belt was assessed: the P 1600 conveyor belt. The results of the mathematical modelling clearly indicate that the created regression models provide a very good description of the real behaviour of a tested rubber-fabric conveyor belt in operations under the dynamic stress due to the effects of the impact force and the tension force. During the measurements carried out with the testing equipment and the drop hammer with a spherical end-piece, only the upper cover layer of a conveyor belt was damaged, whereas the chassis remained undamaged. During the experiment, no disruption of a conveyor belt occurred.

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