



*Mathematical Modelling
and Geometry*

Volume 4, No 1, p. 19 – 33 (2016)

**All the trajectories of an extended averaged
Hebbian learning equation on the quantum state
space are the e-geodesics**

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Abstract. In this paper, two families of trajectories on the quantum state space (QSS) originating from a synaptic-neuron model and from quantum information geometry meet together. The extended averaged Hebbian learning equation (EAHLE) on the QSS developed by the author and Yuya [1] from a Hebbian synaptic-neuron model is studied from a quantum-information-geometric point of view. It is shown that all the trajectories of the EAHLE are the e-geodesics, the autoparallel curves with respect to the exponential-type parallel transport, on the QSS. As a secondary outcome, an explicit representation of solution of the averaged Hebbian learning equation, the origin of the EAHLE, is derived from that of the e-geodesics on the QSS.

Keywords: dynamical systems, quantum information, geodesic, Hebbian learning

MSC numbers: 37D40, 53C22, 68T05, 81P45

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