http://mmg.tversu.ru ISSN 2311-1275

Mathematical Modelling and Geometry

Volume 4, No 1, p. 34 – 40 (2016)

The mathematical model of wax deposition thickness in a pipeline taking the aging of the deposits into account

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Received 8 February 2016, in final form 20 February. Published 22 February 2016.

Abstract.
One of the most common problems in the heavy oil production involves the formation of paraffin wax deposits in pipelines. The inner surface of the pipeline becomes fouled with these paraffin deposits, which reduces the flow diameter, decreases overall through-out, and results in a higher pressure drop when oil is pumped through the pipeline. The deposits within the pipelines decrease the capacity of the duct and cause pipelines breaking. Wax deposition is a serious problem of oil production in the petroleum industry. Therefore, accurate prediction of this solid deposition problem can result in increasing the efficiency and safety of oil production. The authors consider the problem of wax deposition in pipelines and the growth model of paraffin deposits in pipelines which is based on the model developed in the Michigan University. The model describes the time dependence of the deposit growth basing on molecular diffusion. This model also includes the aging of the deposits that is a process of increasing of the wax fraction in the deposit due to the internal diffusion. This research is intended to be a part of the project dealing with the development of the flow simulator. The discussed model is to be integrated in the VSS (Ventilation System Simulator).

Keywords: wax deposition, simulation, pipelines, wax ageing, wax thickness, oil modeling, computational methods
MSC numbers: 76R50,76R99

1. Introduction

Wax deposition is a very complex phenomenon that in recent years is one of the major challenges in oil industry. Wax deposited on the inner surface of crude oil pipelines can reduce or completely stop the oil flow and the oil industry bears large losses (1).

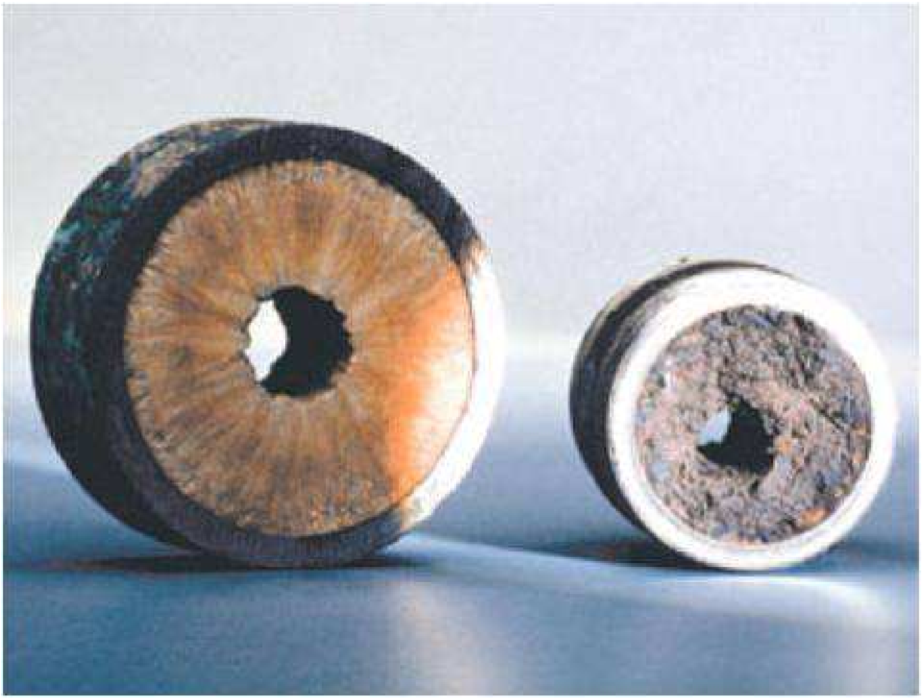

Figure 1: Examples of paraffin deposits in pipelines

The formation of paraffin wax deposition is a complex process, which is influenced by many factors. The main factors are the following:

- *Temperature differences.* The oil in the pipeline interacts with the cooled conduit walls. Two processes begin when temperature reaches the wax appearance point, the crystallization of paraffins in the oil volume and the deposition of paraffins on the cold surface.
- *Influence of flow velocity.* With the increase of petroleum velocity in the conduit the intensity of deposition decreases due to the flushing of depositions with the flow.
- *Conduit wall wettability.* Promotes the appearance of hydrate layers that prevent the deposition.
- *Conduit wall roughness.* Promotes the release of gas that decreases the solubility of paraffin in oil.

The cost for preventing and removing paraffin deposits is substantial. The studies beginning from the early 1990s have shown that this cost amounts to as much as 0.25 of the global GPD [1]. Figure 2 shows the cost of the cleaning procedure for the pipe with the length 15 km, the diameter 0.508 m, and the paraffin wax layer thickness 10 mm.

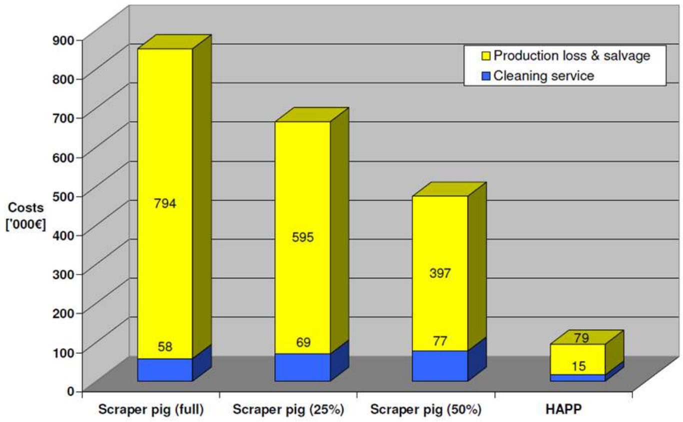

Figure 2: The costs required for procedures of cleaning the pipe with the length 15 km and the paraffin wax layer thickness 10 mm

In order to control the paraffin wax deposition the following processes are generally used:

- Detection of depositions (ultrasonic detection, radioisotope technology, fiber optics technology)
- Prevention of depositions (using of smooth coating, chemical methods: dampening, modifiers, physical methods: vibration, ultrasound, acting with electromagnetic fields)
- Removal of depositions (thermal methods: flushing with hot petroleum, direct steam, electric heater, cold-flow technology, mechanical methods: scrapers, chemical methods: dissolvents)

Predictive modeling of wax deposition has become an indispensable approach, not only to understand the fundamental physics of wax deposition during oil transportation in pipelines, but also to design efficient remediation strategies.

Burger [3] investigated molecular diffusion, Brownian diffusion, shear dispersion and gravity settling as significant physical processes leading to wax deposition in pipelines:

- The molecular diffusion begins when the temperature of the wall reaches WAT (wax appearance temperature). Herewith, the oil is saturated with wax in solution and wax precipitates out. Wax precipitation leads to a concentration gradient between the dissolved wax in the turbulent core and the precipitated wax. The dissolved wax diffuses towards the wall where it forms deposition. The molecular diffusion is described by the modified Fick's law [4].

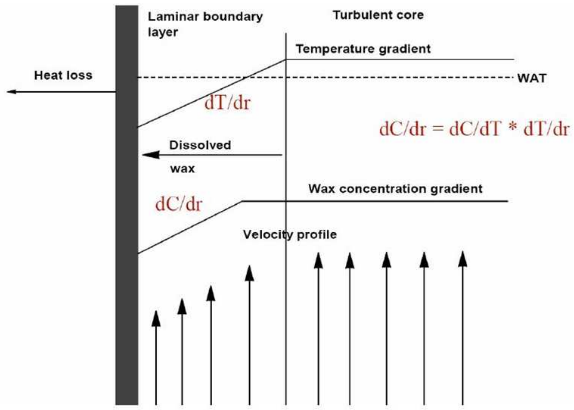

Figure 3: Molecular diffusion as a mechanism of wax deposits forming

- The Taylor shear dispersion is the diffusion process associated with the longitudinal scattering (dispersion) of the solute (impurities) in a straight pipe. The main mechanism of this process is common convective transport in the presence of radial shear flow that interacts with the radial molecular or turbulent diffusion.

- The Brownian diffusion is a result of bombarding wax crystals by thermally excited oil molecules. Due to the concentration gradient, the Brownian motion will lead to particle transport similar to molecular diffusion.

- The gravity settling is a process in which the particulates settle to the bottom of a liquid and form sediment. Particles that experience forces, either due to gravity or due to centrifugal motion will move in a uniform manner in the direction pointed with the force. For gravity settling, this means that the particles will fall to the bottom of the vessel, forming slurry at the vessel base.

The experiments performed at the University of Michigan confirmed that such particulate deposition mechenisms as shear dispersion, Brownian diffusion, and gravity settling are not significant for the conventional flows in a pipeline [3], [4].

Our research is based on the model developed in the University of Michigan [5],[6],[7].The model describes the time dependence of the deposits growth based on the molecular diffusion. This model also includes the ageing of the deposits that is a process of increasing of the wax fraction in the deposit due to the internal diffusion. The coupled system of differential equations is needed to describe the growth and the aging of the deposit:

$$(-2\pi r_{eff})\rho_{wax}F_w\frac{dr_{eff}}{dt} = (2\pi r_{eff})K_M(C_b - C_{wo}(T_i)) - (2\pi r_{eff})\left[-D_e\left.\frac{dC}{dr}\right|_i\right] \quad (1)$$

$$\pi\rho_{wax}(r_i^2 - r_{eff}^2)\frac{dF_w}{dt} = 2\pi r_{eff}\left[-D_e\left.\frac{dC}{dr}\right|_i\right] \quad (2)$$

where $\rho_{wax}(kg/m^3)$ is the density of deposits, $r_i(m)$ is the radius of the clean pipe, $C_{wo}(kg/m^3)$ is the solubility of wax in oil, $F_w(-)$ is the wax content, $\frac{dC}{dr}|_i$ is the concentration gradient at the wall, $\frac{dF_w}{dt}(s^{-1})$ is the rate of the wax fraction in deposit changing with time (aging), $C_b(kg/m^3)$ is the concentration of the wax in the bulk oil, $T_i(K)$ is the pipe inlet temperature and $D_e(m^2/s)$ is the effective diffusivity in the deposit, $r_{eff}(m)$ is the effective flow radius at the time of interest, $K_M(m/s)$ is the mass transfer coefficient.

Thus, the deposit thickness is given by the difference

$$\delta(t) = r_i - r_{eff}(t) \quad (3)$$

The continuity equations for heat and mass transfer, or the heat and mass balance equations, are used to obtain the temperature and concentration gradients in the fluid. In the cylindrical coordinates the governing equations for heat and mass transfer can be written as

$$v_z\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha_T + +\epsilon_H)\frac{\partial T}{\partial r}\right] - \beta(T - T_{wo}) \quad (4)$$

$$v_z\frac{\partial C}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r(D_{wo} + \epsilon_m)\frac{\partial C}{\partial r}\right] - k_r(C - C_{ws}), \quad (5)$$

where $v_z(m/s)$ is the fluid velocity in the axial direction, $T(C)$ is the temperature, $C(kg/m^3)$ is the concentration of wax dissolved in oil, $z(m)$ is the axial distance, $r(m)$ is the radial position, ϵ_H and $\epsilon_m(m^2/s)$ are the turbulent heat and mass diffusivities, $\alpha_T(m^2/s)$ is the turbulent heat diffusivity, $D_{wo}(m^2/s)$ is the binary diffusion coefficient of wax in oil, $\beta(s^{-1})$ is the crystallization constant for the heat of fusion, and $k_r(s^{-1})$ is the precipitation rate constant.

Results and Discussion

During this research work the authors performed the simulation of the deposits growth in the pipe. The properties used in the modeling are presented in the Table 1: The results of our calculations are presented in Figs. 4 and 5. They are in good agreement with those obtained in the Michigan University.

The formation of paraffin wax deposition is a complex process, which is influenced by many factors: temperature differences, pressure changes, flow velocity, oil-water ratio,conduit wall wettability and roughness, composition of the oil etc. The authors checked the influence of the velocity on the process of wax deposition. The results are shown in Fig. 4:

This research work is planned to be a part of the project devoted to the development of the flow simulator and the discussed model is to be integrated in the SMTVSS. In this simulator the complex network of pipelines is represented as a

Table 1: The parameters of the model used in the simulation

Pipe properties	length: 2.44 m; inner radius: 0.0072 m; wall temperature: 8.3°C
Oil properties	density: 900 kg/m^3; thermal conductivity: 0.149 W/(mK); specific heat: 1986 J/(kgK); wax content: 0.67 wt
Paraffin properties	density: 970 kg/m^3
Oil flow properties	velocity: 0.2 m/s; temperature: 22.8°C

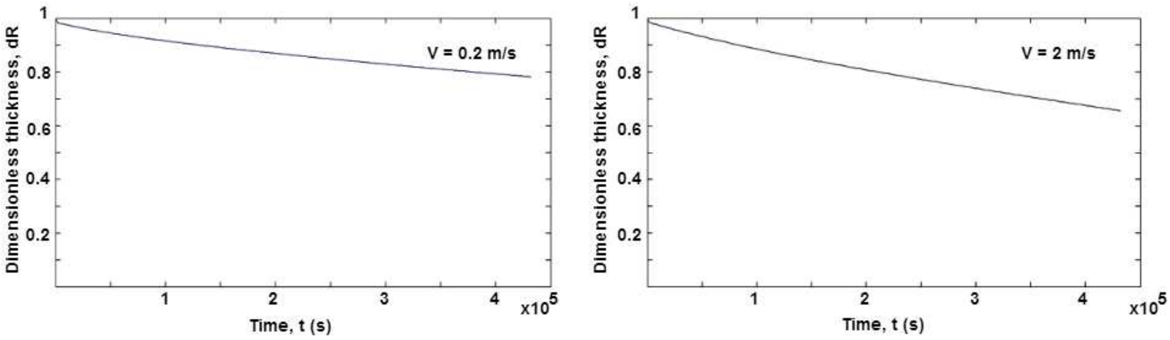

Figure 4: Dimensionless internal radius of the pipe versus time for two values of the flow velocity

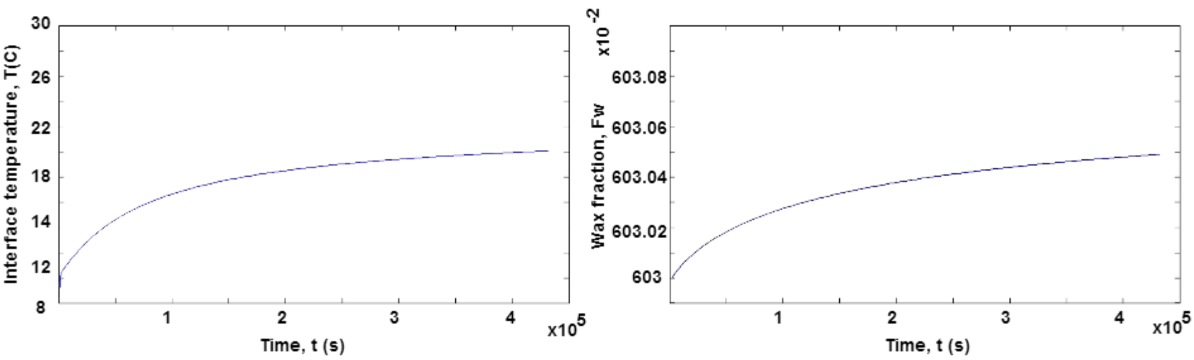

Figure 5: Interface temperature and wax content versus time

graph. This representation enables one to write mass and energy conservation laws in terms of the graph components using the rules, known as Kirchhoff laws, and thus simplify the solution of the flow distribution problem [8].

The integration of WALTER into SMTVSS offers the possibility of fast calculation of flow distribution in a complex network of pipelines, followed by more detailed calculation of wax deposition growth in particular pipelines, where this growth is most likely to occur.

Acknowledgements. The financial support for this study was provided by Siemens, Corporate Technologies. The authors would like to thank I.Nikolin and D.Mustafina for their helpful suggestions and technical assistance during the research, and Prof. S.Yu.Slavyanov, Physical Faculty St.Petersburg State University, for valuable comments that greatly improved the manuscript.

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