



## Quasiclassical Two-Coulomb-Centre Wave Functions in the Spheroidal Coordinate System

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**Abstract.** The consistent scheme for obtaining WKB expansions for solutions of the quasiaangular equation in quantum mechanical problem of two Coulomb centers  $Z_1eZ_2$  is elaborated. It is shown that in each order of  $1/R$  the corrections to the wave function can be expressed by a finite number of the discrete spectrum Coulomb wave functions with modified charge. The simple analytical expressions for the first and second corrections to the wave functions are obtained.

**Keywords:** quasiclassical approximation, WKB method, two Coulomb centers, asymptotic solutions

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