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Discrete dynamical models: combinatorics, statistics and continuum approximations

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Abstract. This essay advocates the view that any problem that has a meaningful empirical content, can be formulated in constructive, more definitely, finite terms. We consider combinatorial models of dynamical systems and approaches to statistical description of such models. We demonstrate that many concepts of continuous physics — such as continuous symmetries, the principle of least action, Lagrangians, deterministic evolution equations — can be obtained from combinatorial structures as a result of the large number approximation. We propose a constructive description of quantum behavior that provides, in particular, a natural explanation of appearance of complex numbers in the formalism of quantum mechanics. Some approaches to construction of discrete models of quantum evolution that involve gauge connections are discussed.

Keywords: combinatorial models, quantum mechanics, finite groups, gauge invariance, statistical descriptions

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